The Dawn Discovery Mission to Vesta and Ceres: Optimal Control of Spaceflight

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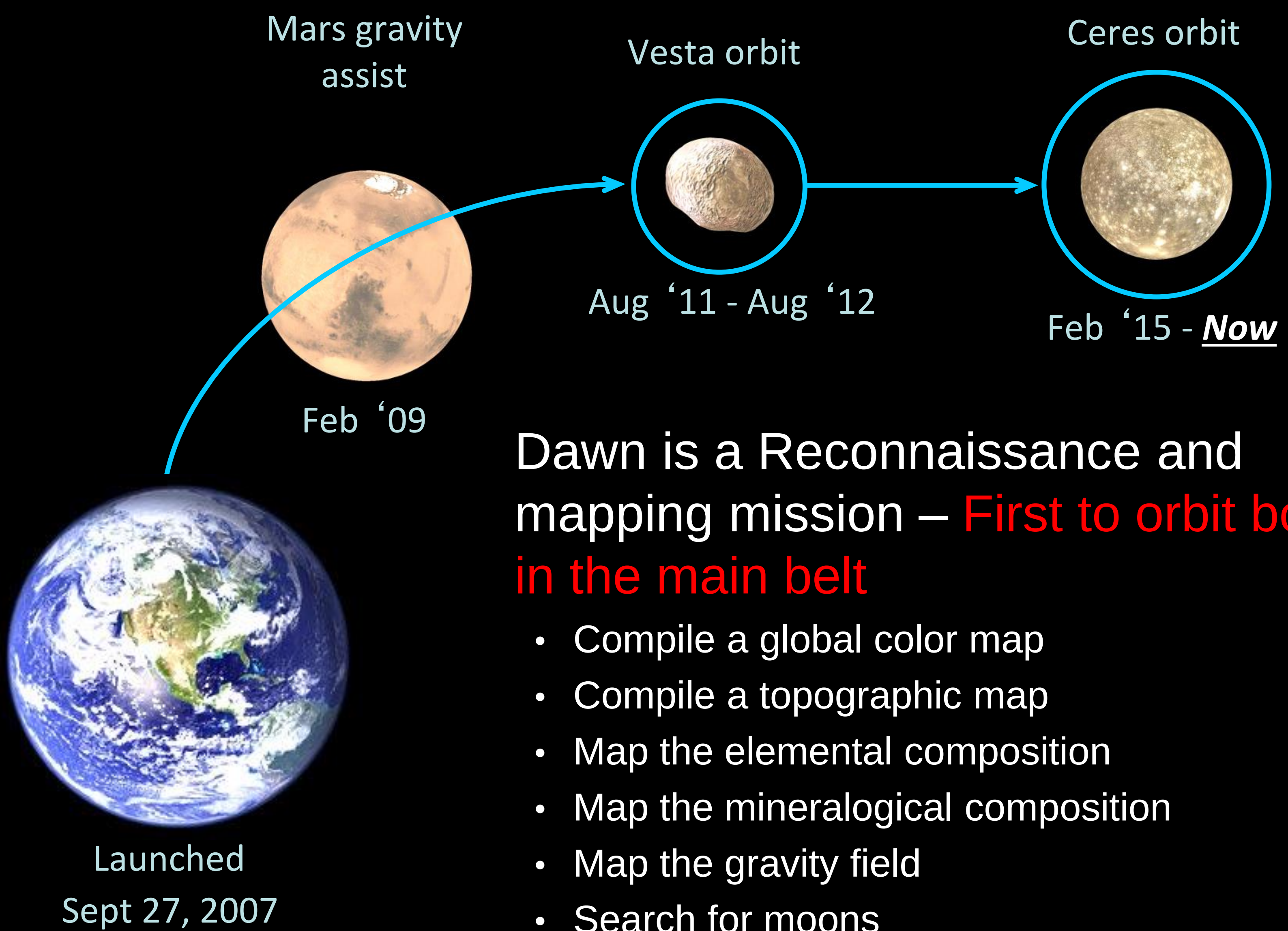
7/6/2017

Dawn Discovery Mission

- The name “Dawn” refers to the “Dawn of the solar system” when the formation of all the planets we know today (and a few we can only infer or don’t know about!) came into existence.

- By visiting the two largest bodies between Mars and Jupiter we see early planet building frozen in time.

Dawn Mission Itinerary



Dawn is a Reconnaissance and mapping mission – First to orbit bodies in the main belt

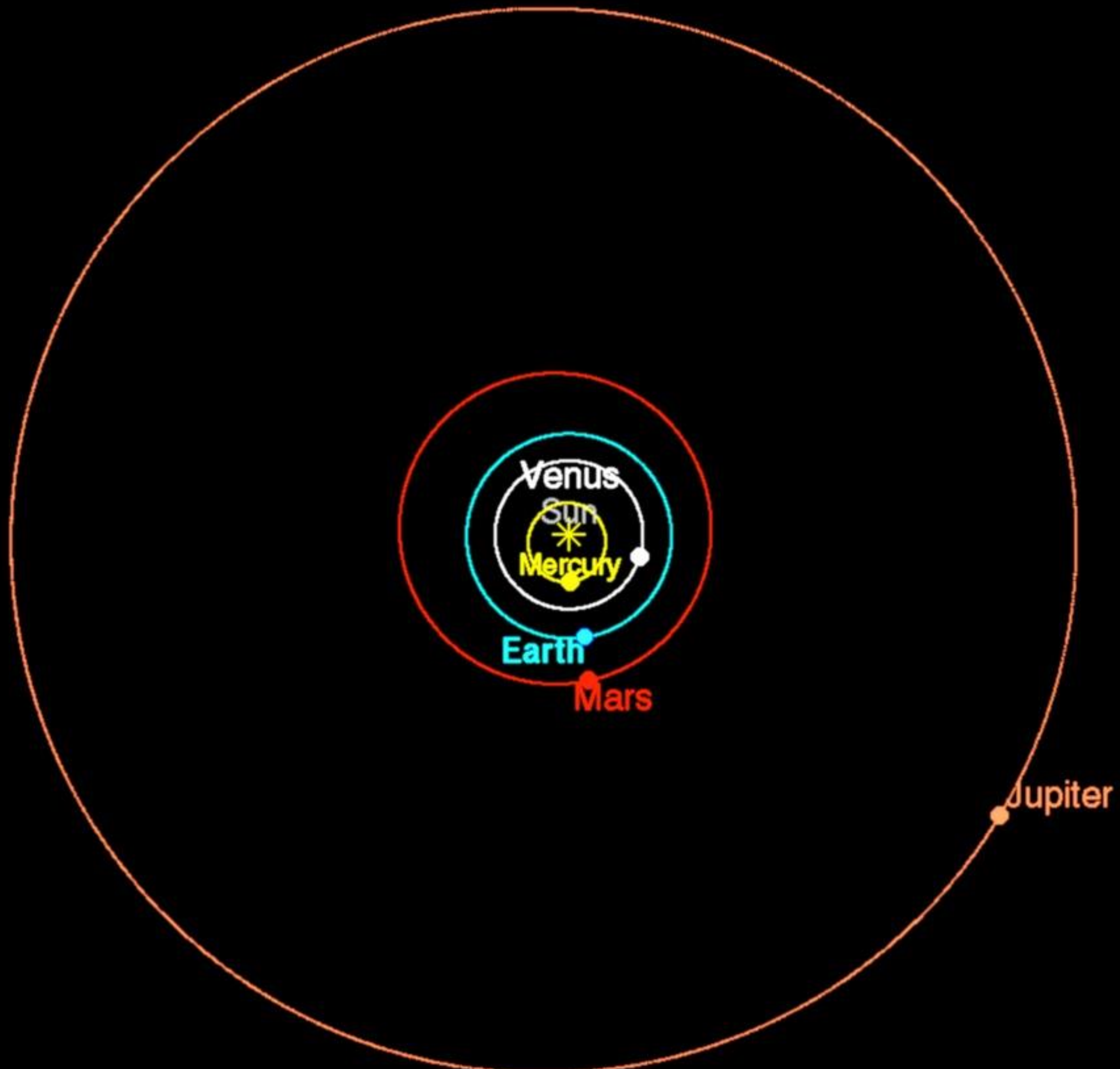
- Compile a global color map
- Compile a topographic map
- Map the elemental composition
- Map the mineralogical composition
- Map the gravity field
- Search for moons

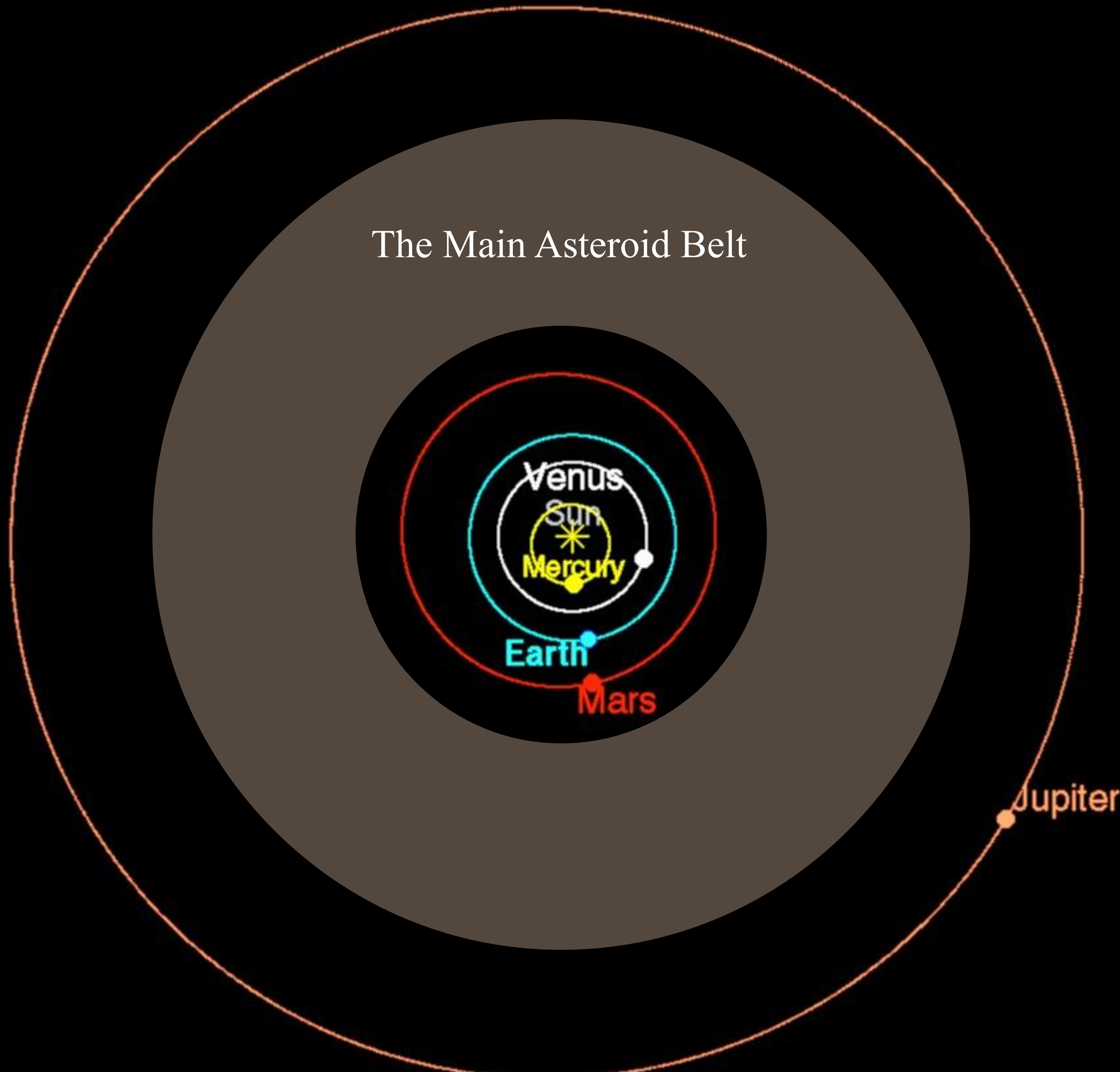
Dawn Discovery Mission

Why Vesta and Ceres?

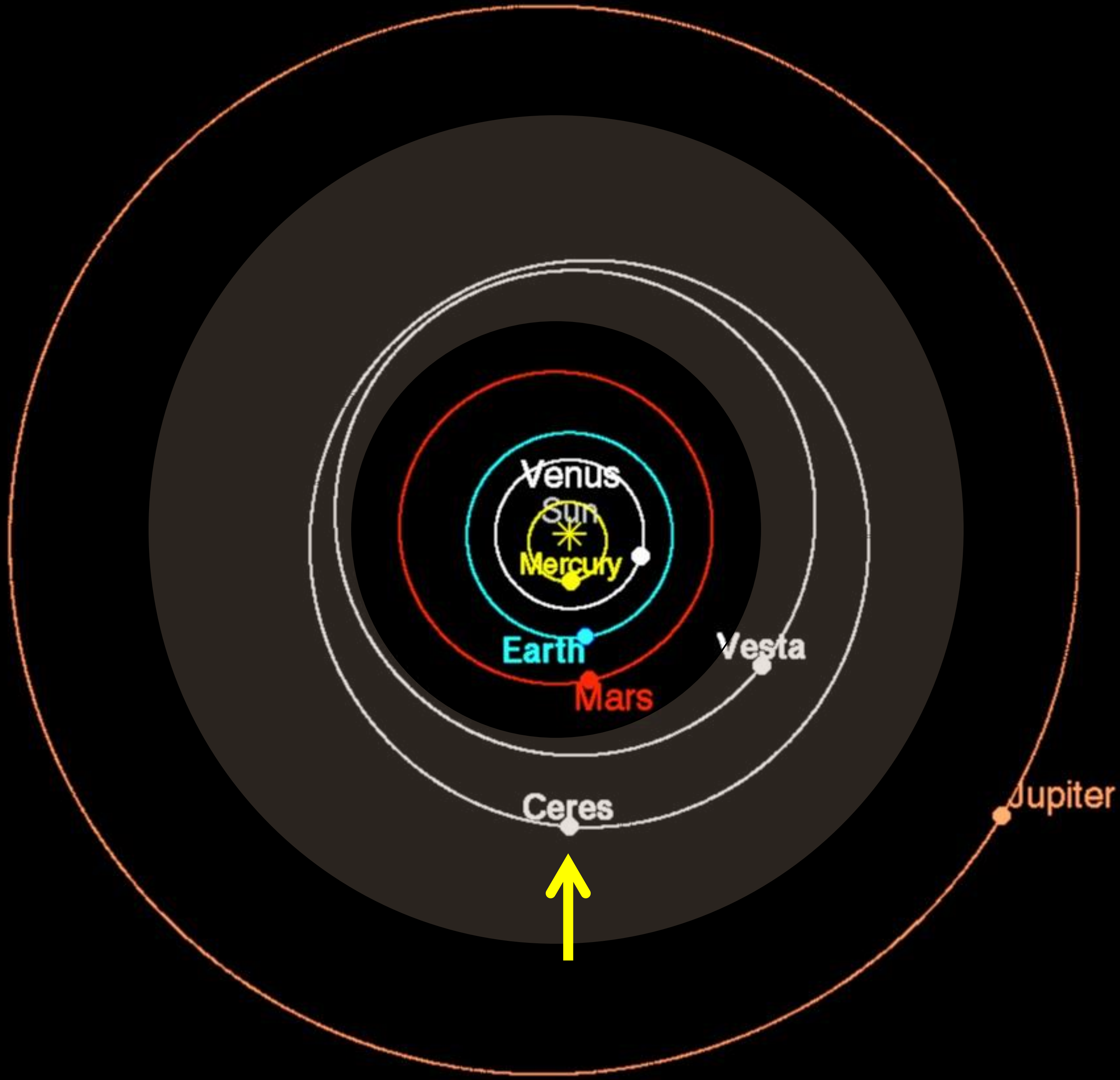
- By visiting Vesta and Ceres we have visited 50% of the mass of the entire asteroid belt. These two are the giants in this region of space.
- Both are planetary embryos – their development ended when Jupiter formed stripping the region of all planet building matter.
    - Vesta is a rocky world like Mercury, Venus, Earth and Mars
    - Ceres is an ice world like the moons of the giant outer planets and possibly like the Kuiper Belt bodies (Pluto)

Where?



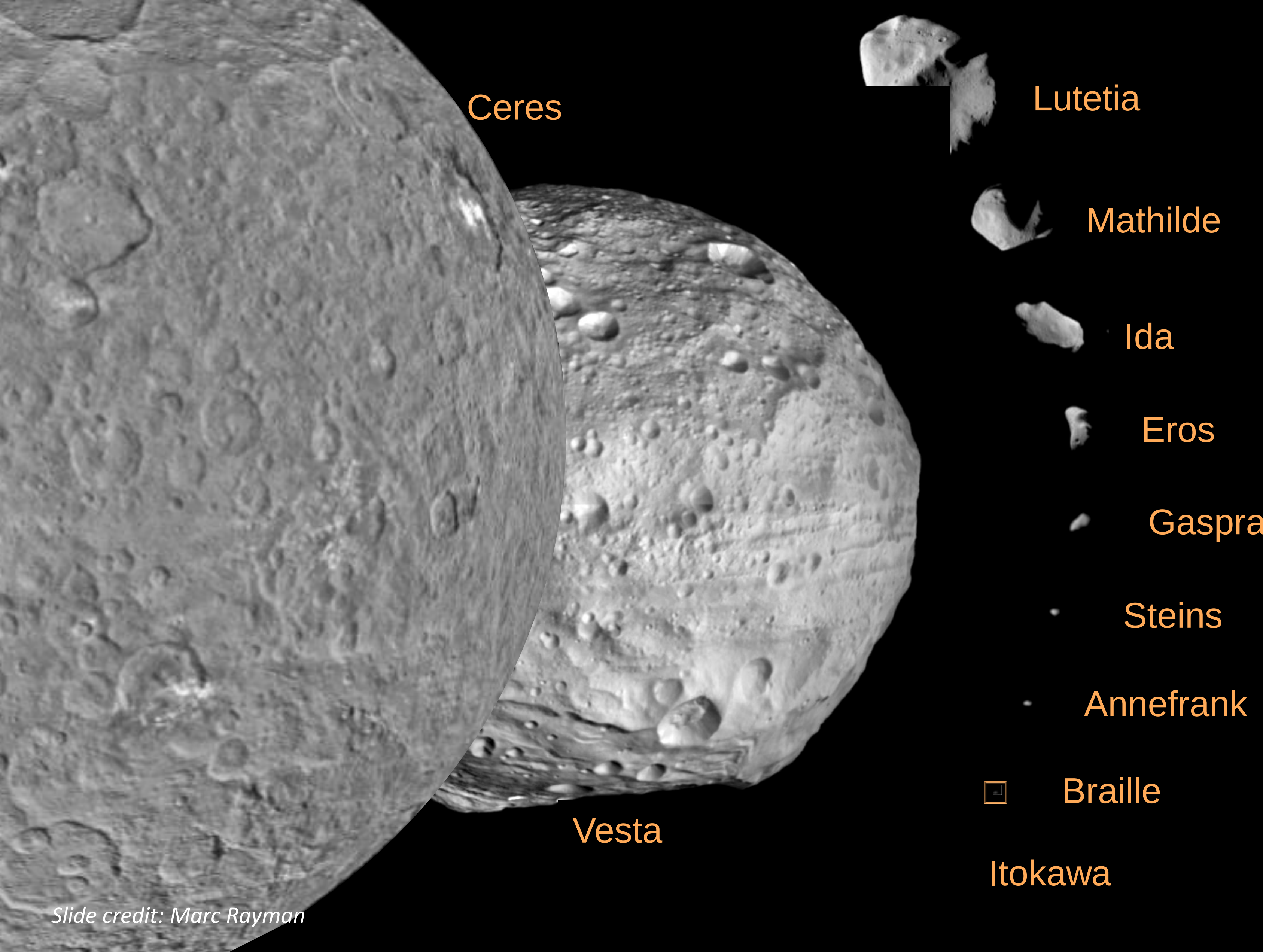

The Main Asteroid Belt
Venus
Sun
Mercury
Earth
Mars
Jupiter

We orbited **Vesta** and are now orbiting **Ceres**



Ceres Size in Context

Vesta Size in Context



Slide credit: Marc Rayman

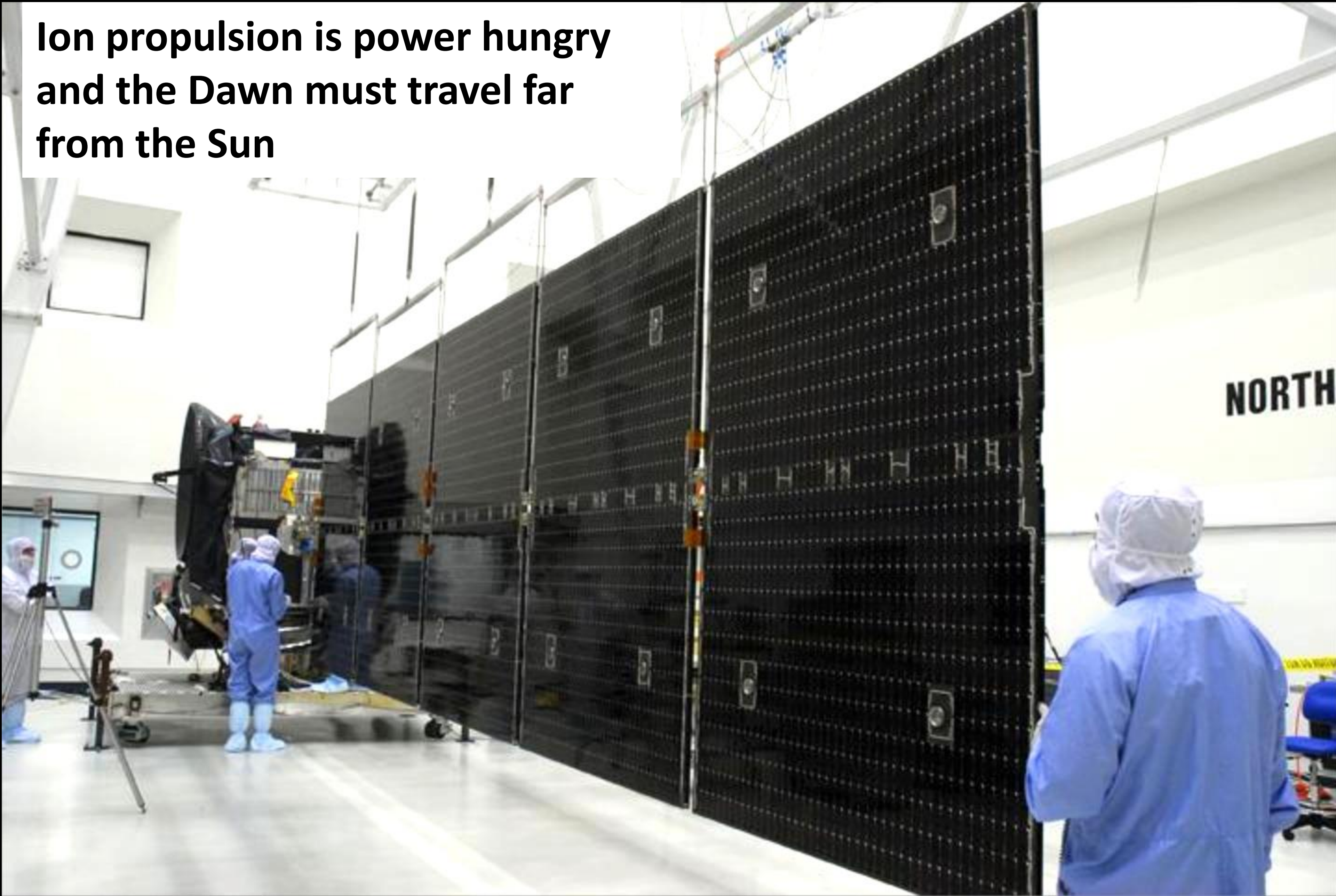
The 1,200 Pounds of Xenon and Ion Engines

Can Accelerate the Dawn spacecraft Again As Much As

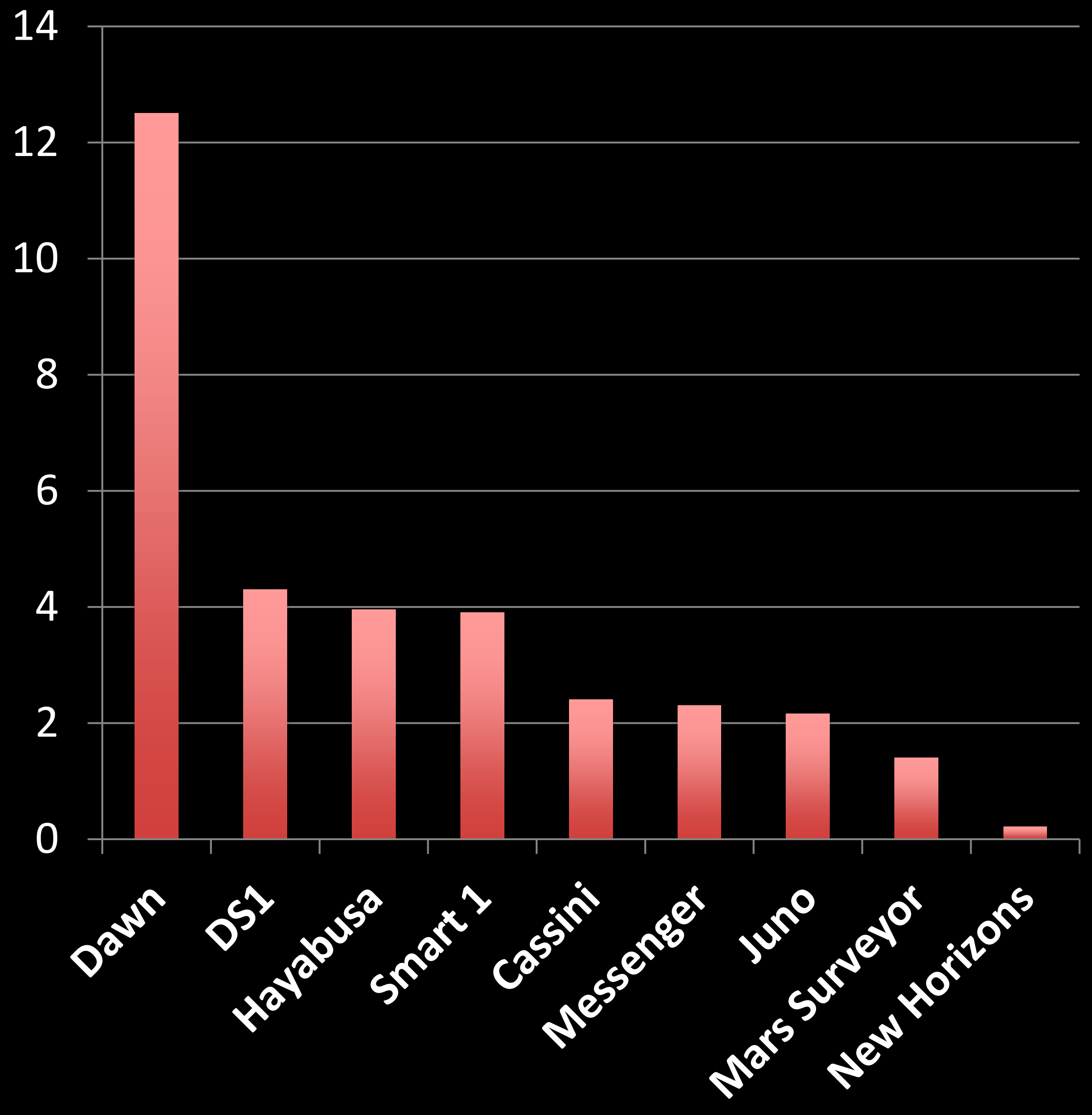
The 700,000 Pound Delta II Rocket Did During Launch.

Dawn is necessarily quite large because of its solar panels

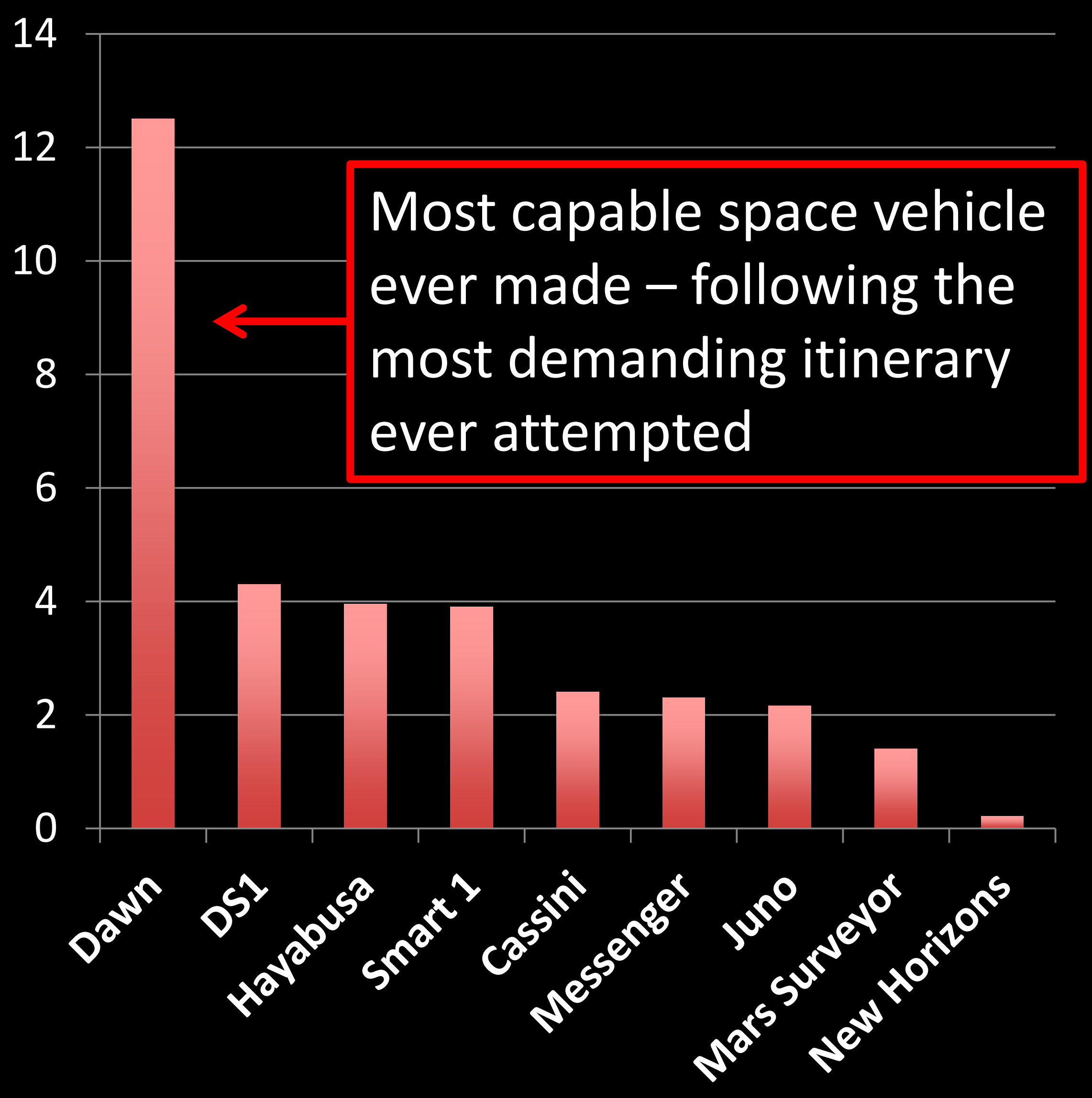
Ion propulsion is power hungry and the Dawn must travel far from the Sun



Post Launch Spacecraft Propulsive capability measured in units of kilometers per second



Post Launch Spacecraft Propulsive capability measured in units of kilometers per second



We Began Our 6 Billion Kilometer Journey On Top A Rocket in 2007

Launch Vehicle

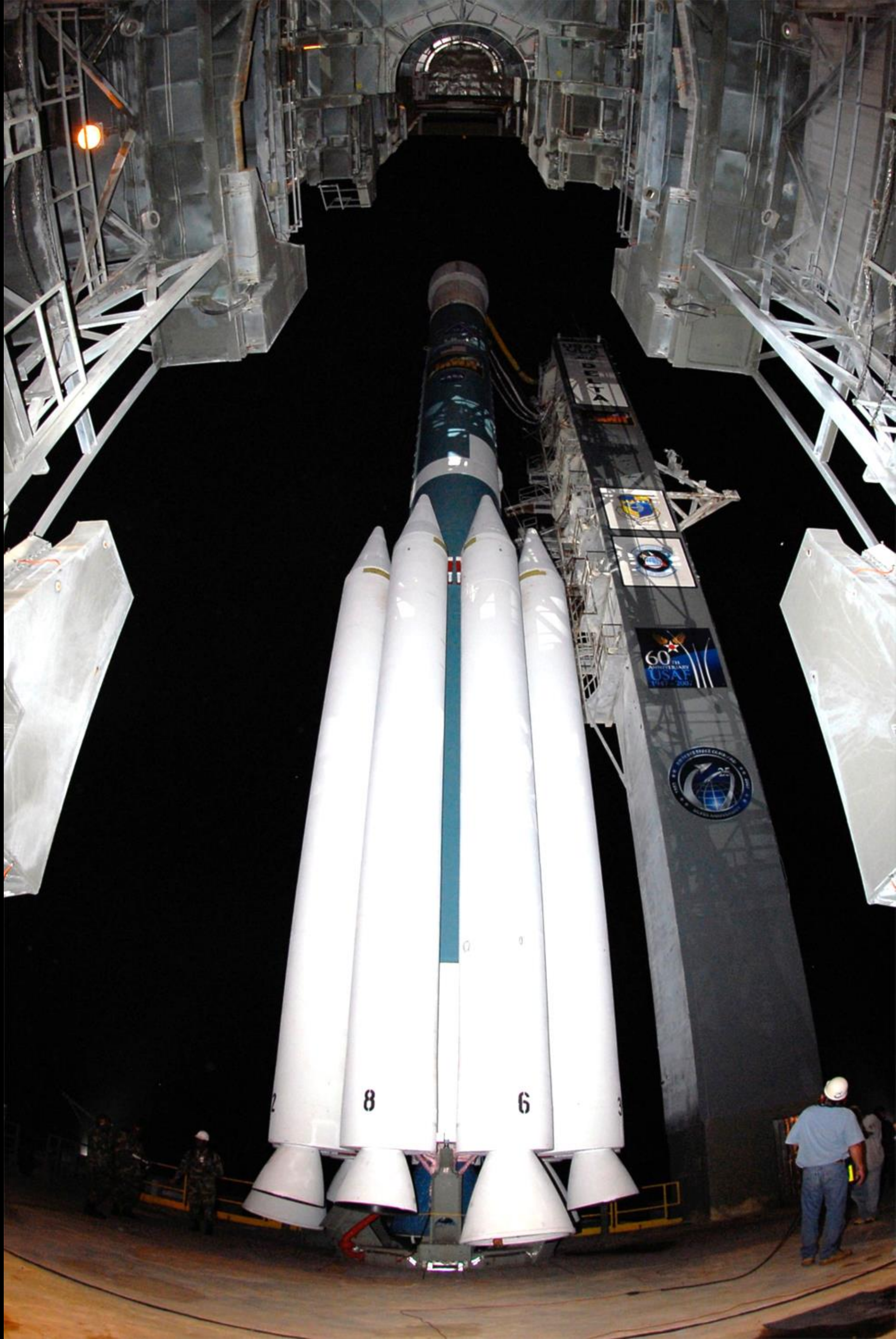
The Delta II Heavy

It started Dawn on its way at 41,000 km/hour away from the Earth on September 27, 2007.

After that, Dawn's **ion engines** take over.

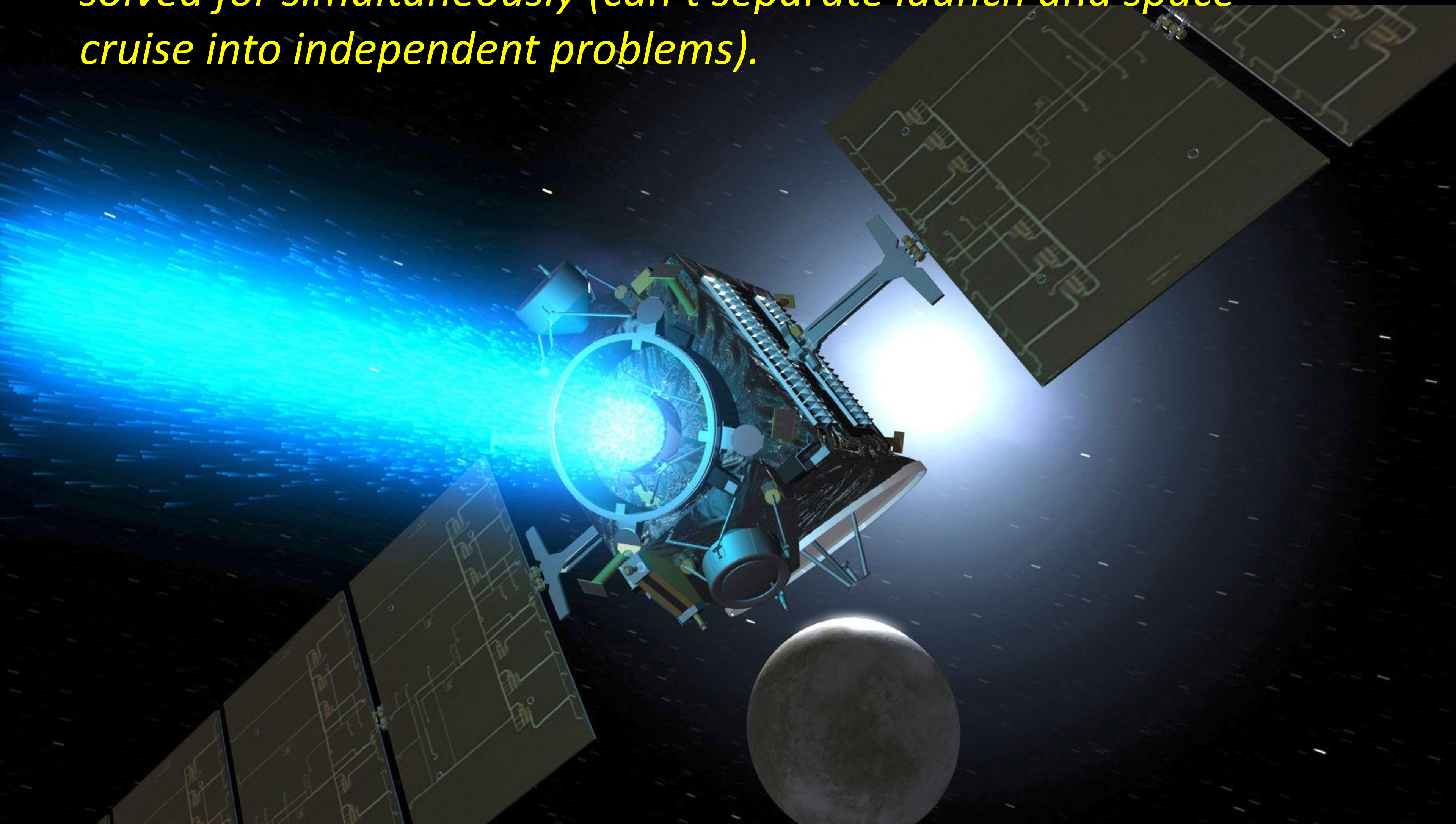
My job was to aim this rocket and then aim the ion engines during the long cruises and orbital operations at Vesta and Ceres

ALL DONE USING OPTIMAL CONTROL TO SOLVE AN END TO END PROBLEM THAT MAXIMIZES THE "DRY" SPACECRAFT MASS DELIVERED TO CERES



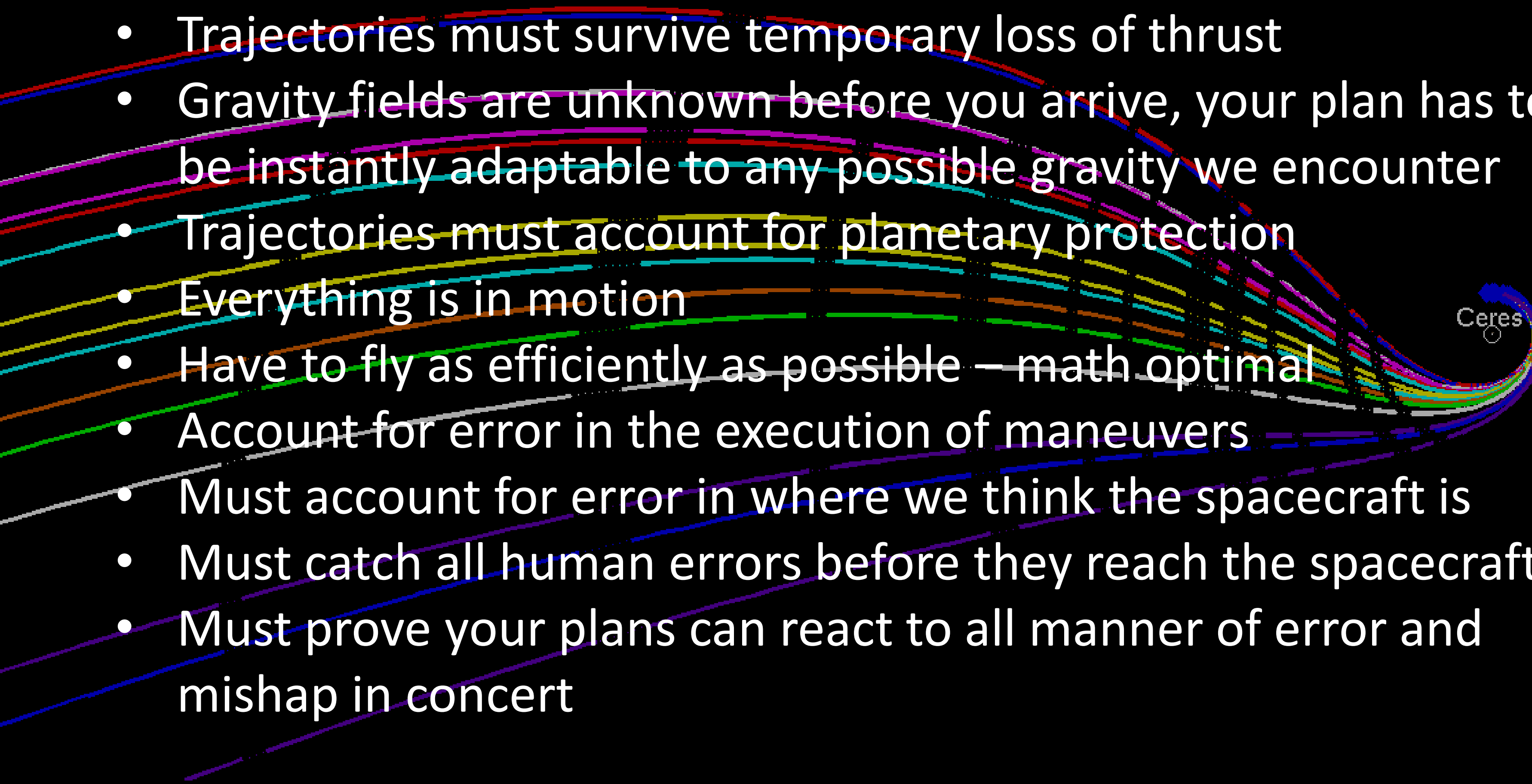
Dawn Launched into the Dawn on September 27, 2007

From an optimal control standpoint, the controls for launch – very high thrust, and the controls for ion propulsion - low-thrust and high efficiency are very different. However, both have to be solved for simultaneously (can't separate launch and space cruise into independent problems).



What is so challenging and interesting about trajectory design?

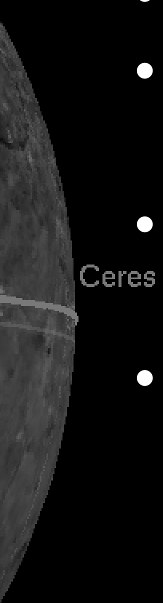
- No way to repair the spacecraft after launch – need redundancy for all systems, and careful load balancing
- Trajectories must survive temporary loss of thrust
- Gravity fields are unknown before you arrive, your plan has to be instantly adaptable to any possible gravity we encounter
- Trajectories must account for planetary protection
- Everything is in motion
- Have to fly as efficiently as possible – math optimal
- Account for error in the execution of maneuvers
- Must account for error in where we think the spacecraft is
- Must catch all human errors before they reach the spacecraft!
- Must prove your plans can react to all manner of error and mishap in concert

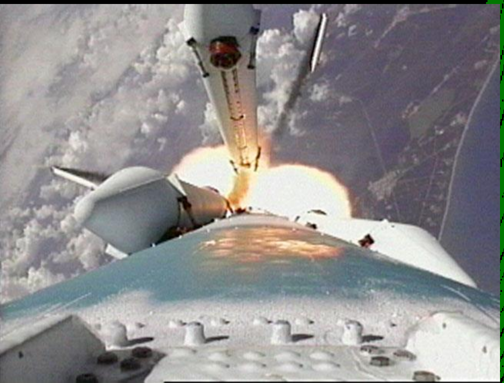


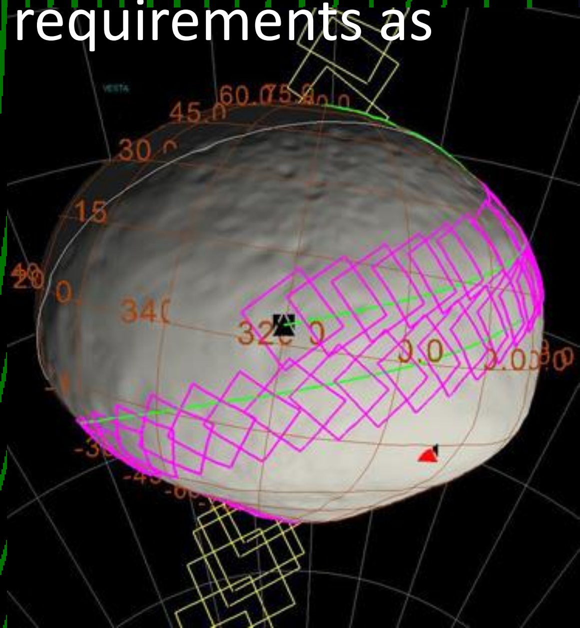
And this is just the beginning!

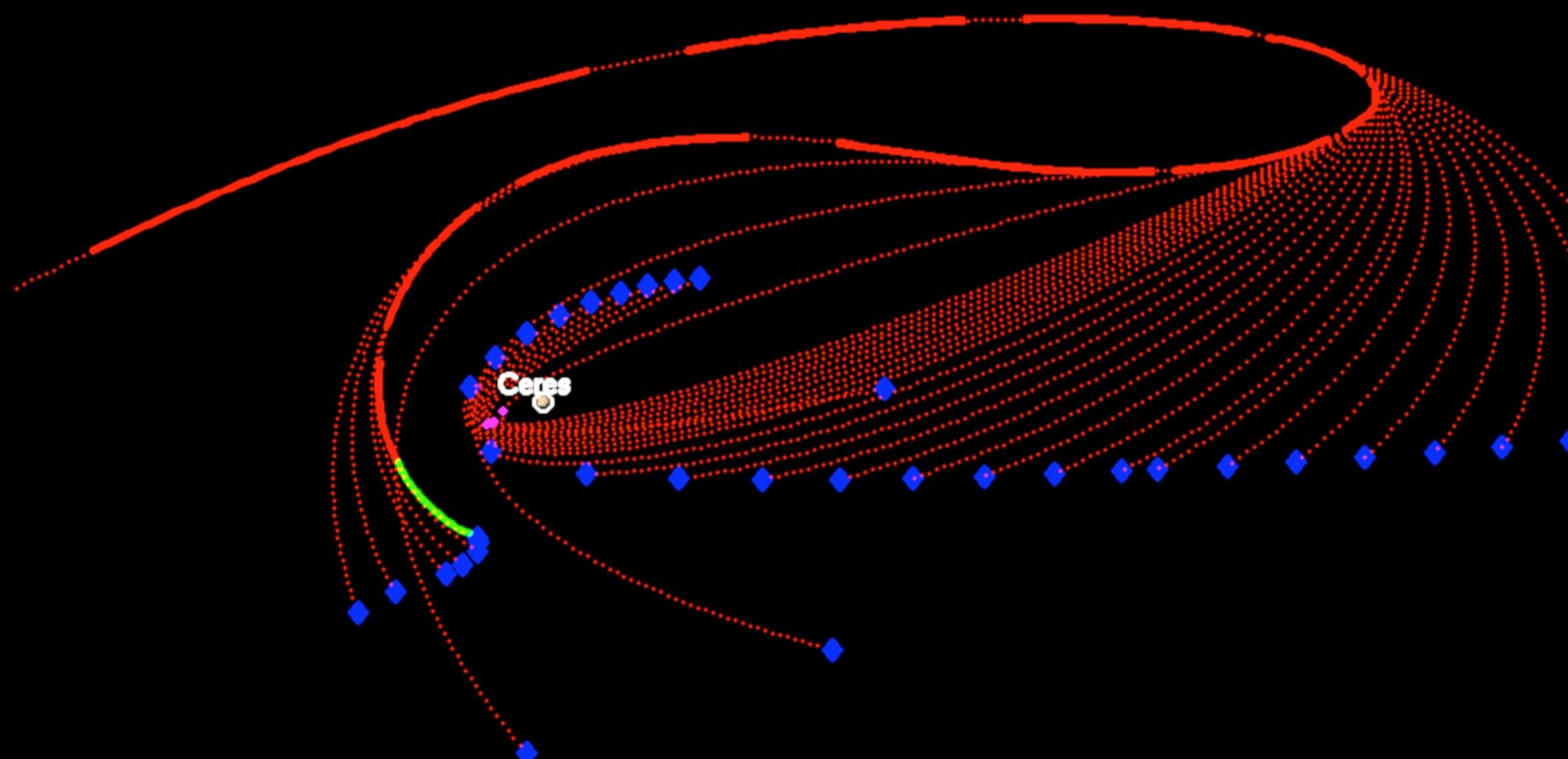
What is so challenging and interesting about trajectory design?

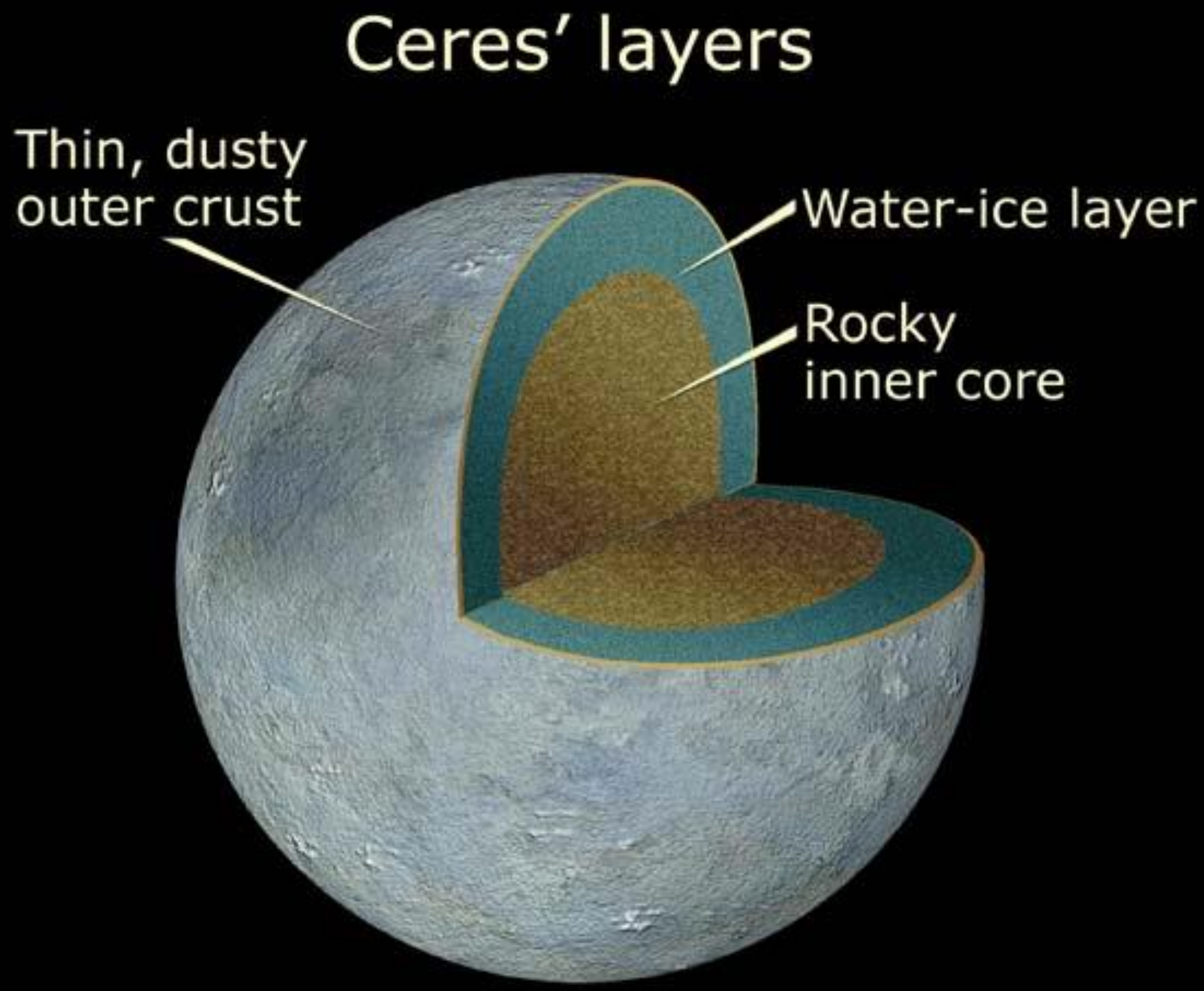
- Multi-body dynamics
- Solar radiation pressure
- Relativity (orbit determination)
- Non-spherical (rotating) gravity fields to surf in
- In flight redesigns are common – things break you have to solve wholly unanticipated problems – must innovate
- Messy real world constraints you'll will not find in academic treatments
- Keeping the scientists happy (ever changing requirements as we explore a new world)








Ceres



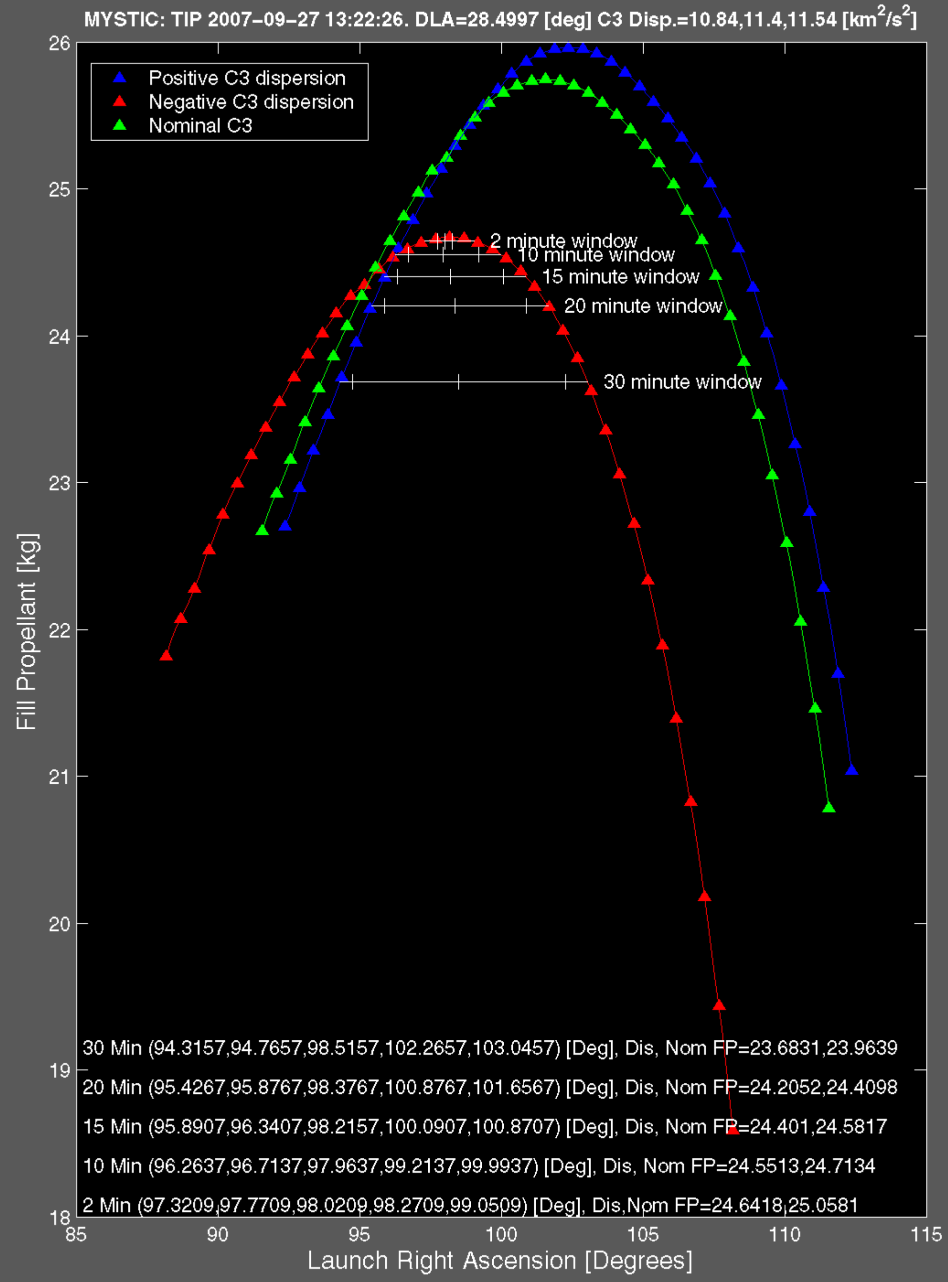
NASA, ESA, and A. Field STScI

And it all starts on the launch pad: One giant coupled problem.

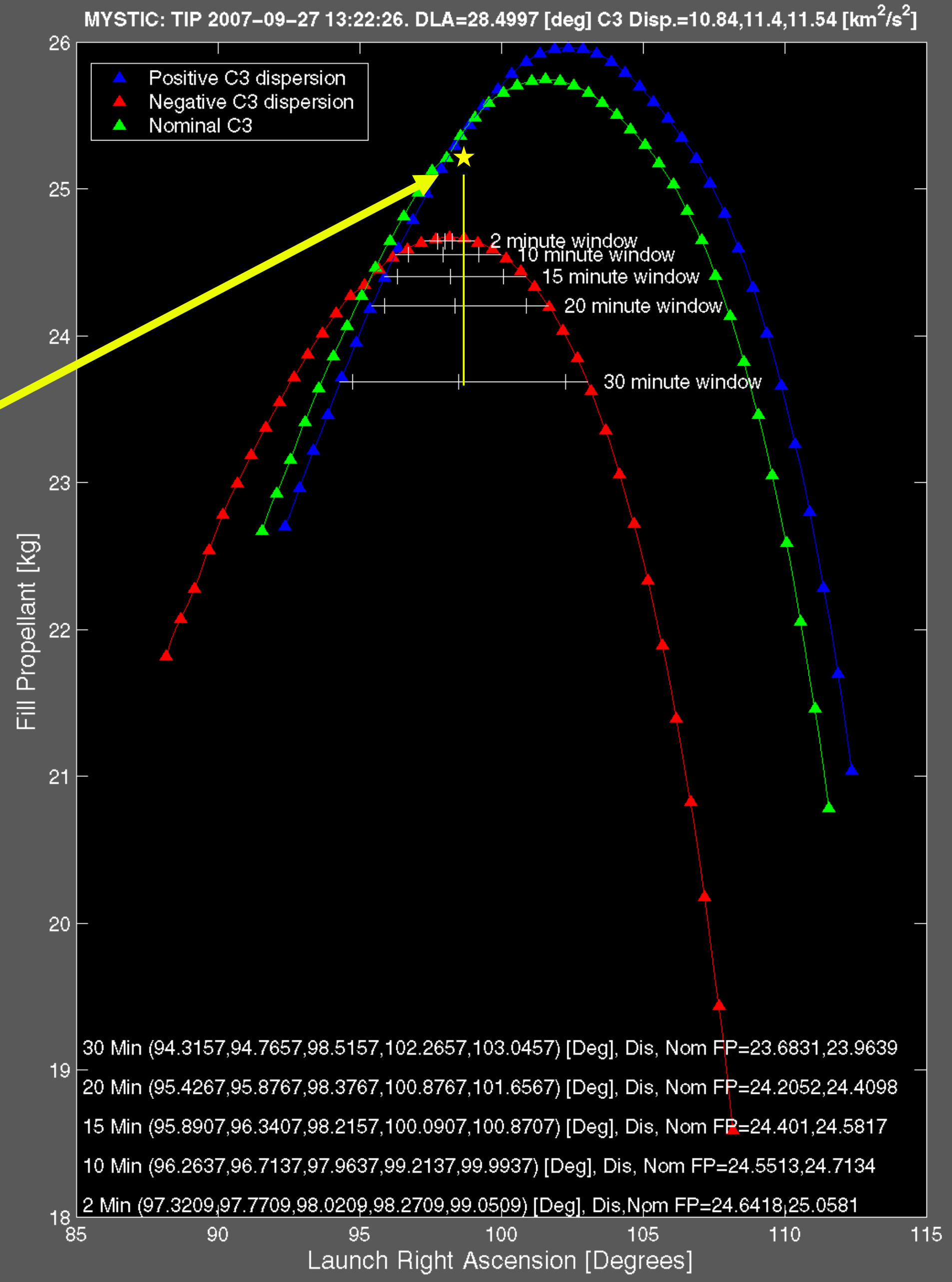
Each point represents an optimal trajectory all the way to Ceres with fixed launch direction and energy

Different colors represent the nominal and extremes of what the launch vehicle may do

This is September 27, 2007 the day we actually launched

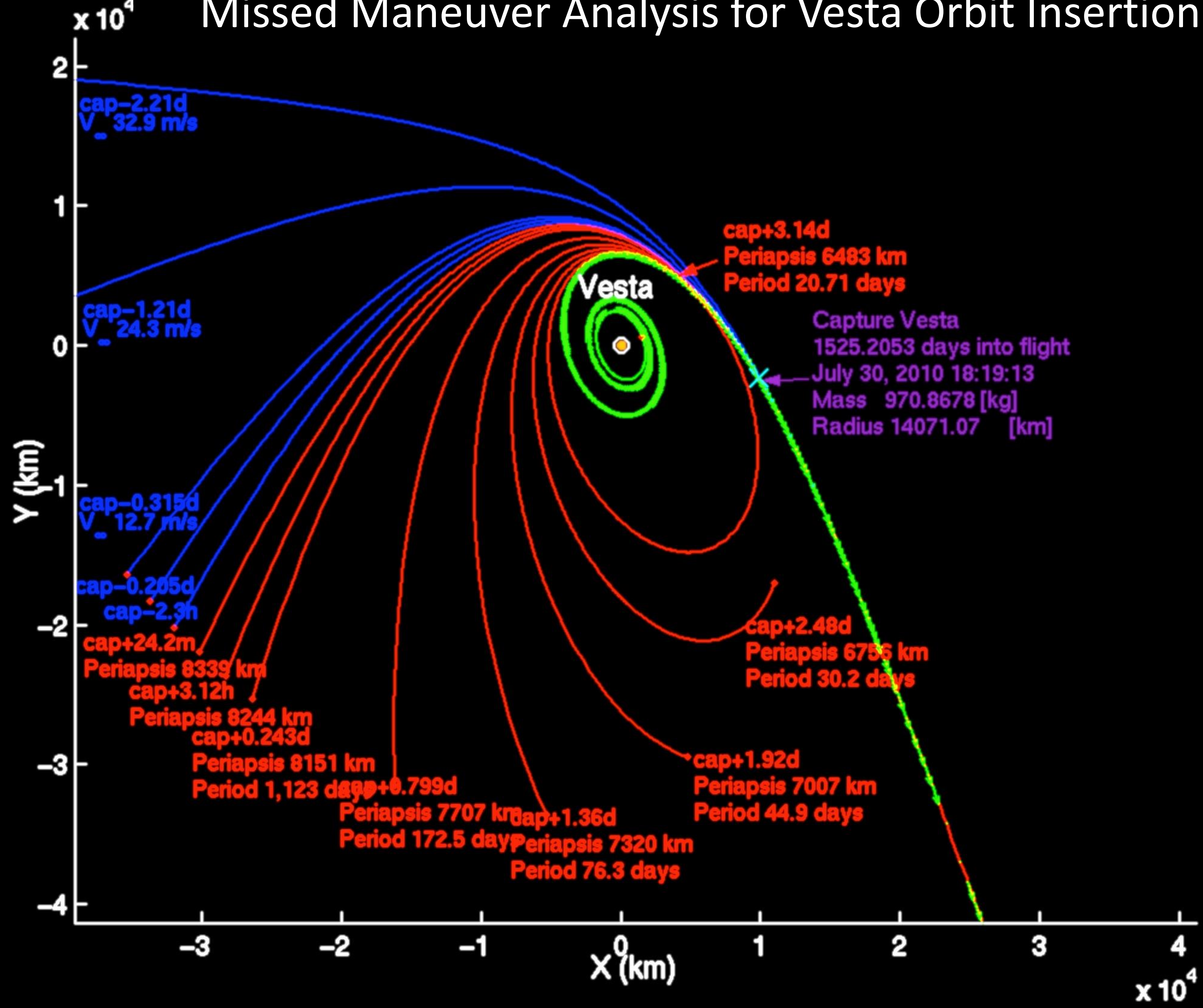


What We Got:

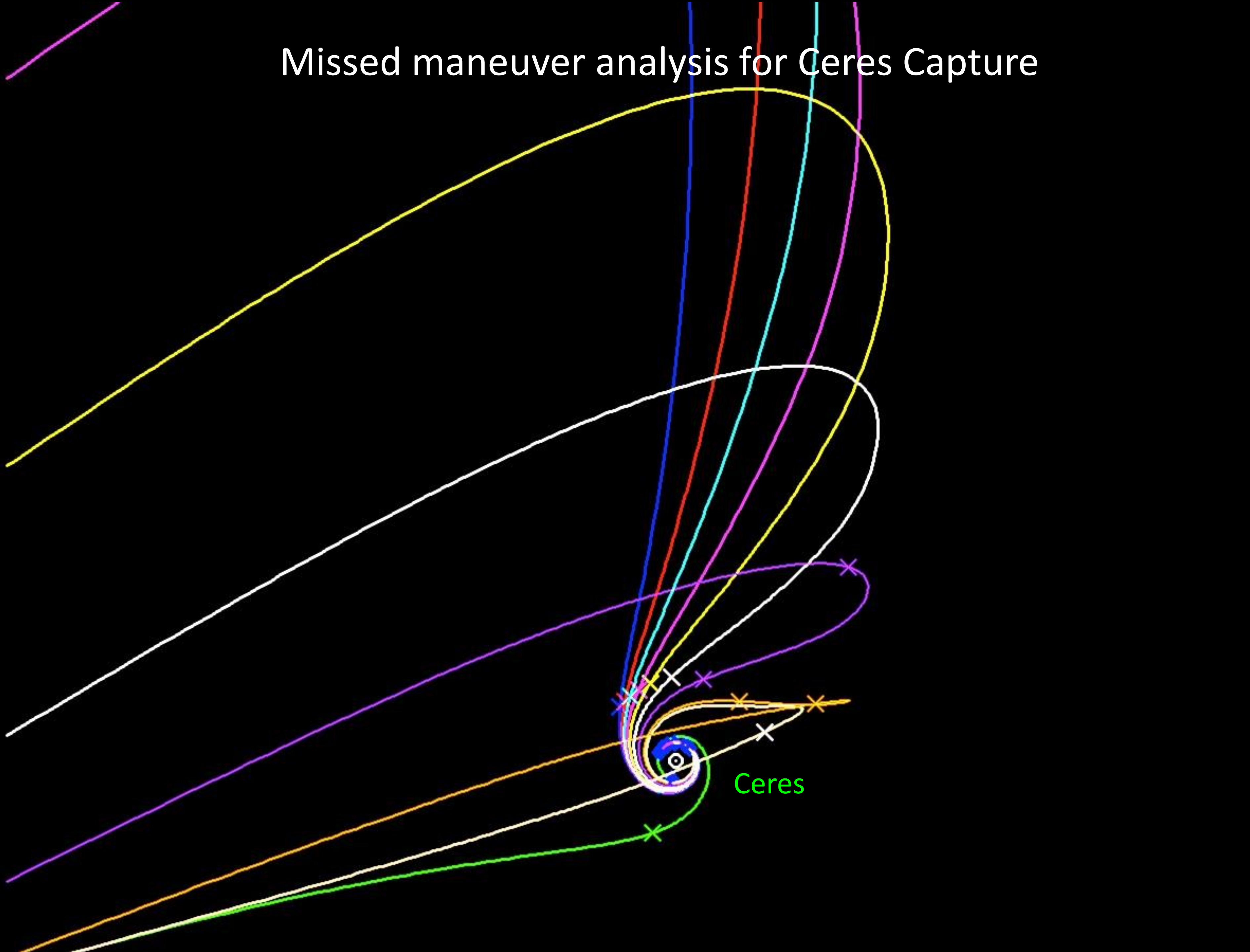


MYSTIC: TIP 2007–09–27 13:22:26. DLA=28.4997 [deg] C3 Disp.=10.84,11.4,11.54 $[km^2/s^2]$
Positive C3 dispersion
Negative C3 dispersion
Nominal C3
2 minute window
10 minute window
15 minute window
20 minute window
30 minute window
30 Min (94.3157,94.7657,98.5157,102.2657,103.0457) [Deg], Dis, Nom FP=23.6831,23.9639
20 Min (95.4267,95.8767,98.3767,100.8767,101.6567) [Deg], Dis, Nom FP=24.2052,24.4098
15 Min (95.8907,96.3407,98.2157,100.0907,100.8707) [Deg], Dis, Nom FP=24.401,24.5817
10 Min (96.2637,96.7137,97.9637,99.2137,99.9937) [Deg], Dis, Nom FP=24.5513,24.7134
2 Min (97.3209,97.7709,98.0209,98.2709,99.0509) [Deg], Dis,Nom FP=24.6418,25.0581
Fill Propellant [kg]
Launch Right Ascension [Degrees]
26
25
24
23
22
21
20
19
18
85
90
95
100
105
110
115

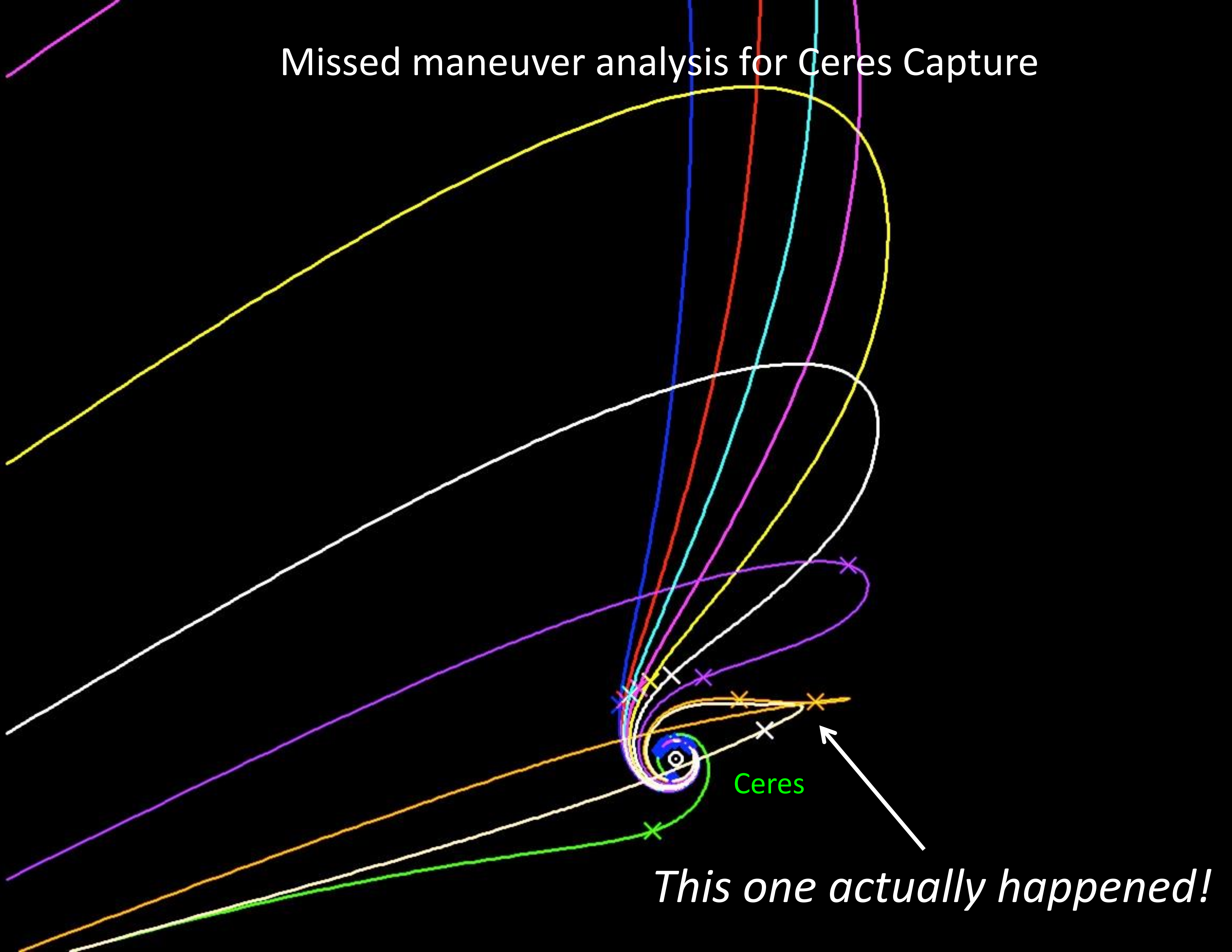
Missed Maneuver Analysis for Vesta Orbit Insertion



Missed maneuver analysis for Ceres Capture



Missed maneuver analysis for Ceres Capture



State:

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ x_6(t) \\ x_7(t) \end{bmatrix} = \begin{bmatrix} x\ coordinate\ of\ spacecraft \\ y\ coordinate\ of\ spacecraft \\ z\ coordinate\ of\ spacecraft \\ x\ velocity\ of\ spacecraft \\ y\ velocity\ of\ spacecraft \\ z\ velocity\ of\ spacecraft \\ mass\ of\ the\ spacecraft. \end{bmatrix}$$

Dynamic control:

$$v(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix} = \begin{bmatrix} x\ component\ of\ thrust \\ y\ component\ of\ thrust \\ z\ component\ of\ thrust. \end{bmatrix}$$

Parameters ("static control"):

$$w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \\ w_8 \\ w_9 \\ w_{10} \\ w_{11} \end{bmatrix} = \begin{bmatrix} date\ of\ trajectory\ start \\ total\ flight\ time \\ longitude\ of\ the\ ascending\ node \\ argument\ of\ the\ periapsis \\ true\ anomaly \\ Orbital\ C_3 \\ Periapsis\ radius \\ inclination \\ initial\ mass \\ thuster\ specific\ impulse \\ solar\ array\ size \end{bmatrix}$$

The optimal Control problem:

Objective: $$maximize_{v(t),w} \; (Spacecraft\ final\ mass)$$

x(t): spacecraft state v(t): thrust vector w: parameters

The optimal Control problem:

Objective: $$maximize_{v(t),w} \ (Spacecraft\ final\ mass)$$

State equation: $$\frac{dx(t)}{dt} = T(x(t), v(t), w, t)$$ *Physics*

x(t): spacecraft state v(t): thrust vector w: parameters

The optimal Control problem:

Objective: $$maximize_{v(t),w} \ (Spacecraft\ final\ mass)$$

State equation: $$\frac{dx(t)}{dt} = T(x(t), v(t), w, t)$$

Initial Condition: $$x(t_0) = \Gamma(w)$$ *Physics/Semi-controllable*

$x(t)$: spacecraft state $v(t)$: thrust vector w: parameters

The optimal Control problem:

Objective: $$maximize_{v(t),w} \ (Spacecraft\ final\ mass)$$

State equation: $$\frac{dx(t)}{dt} = T(x(t), v(t), w, t)$$

Initial Condition: $$x(t_0) = \Gamma(w)$$

Target Final State: $$\Psi(x(t_f), v(t_f), w, t_f) = \ or \ \leq k_1$$ *Arrive at Ceres*

x(t): spacecraft state v(t): thrust vector w: parameters

The optimal Control problem:

Objective: $maximize_{v(t),w} \; (Spacecraft \; final \; mass)$

State equation: $$\frac{dx(t)}{dt} = T(x(t), v(t), w, t)$$

Initial Condition: $$x(t_0) = \Gamma(w)$$

Target Final State: $$\Psi(x(t_f), v(t_f), w, t_f) = \; or \; \leq k_1$$

Intermediate State: $$\Psi(x(t_i), v(t_i), w, t_i) = \; or \; \leq k_i$$

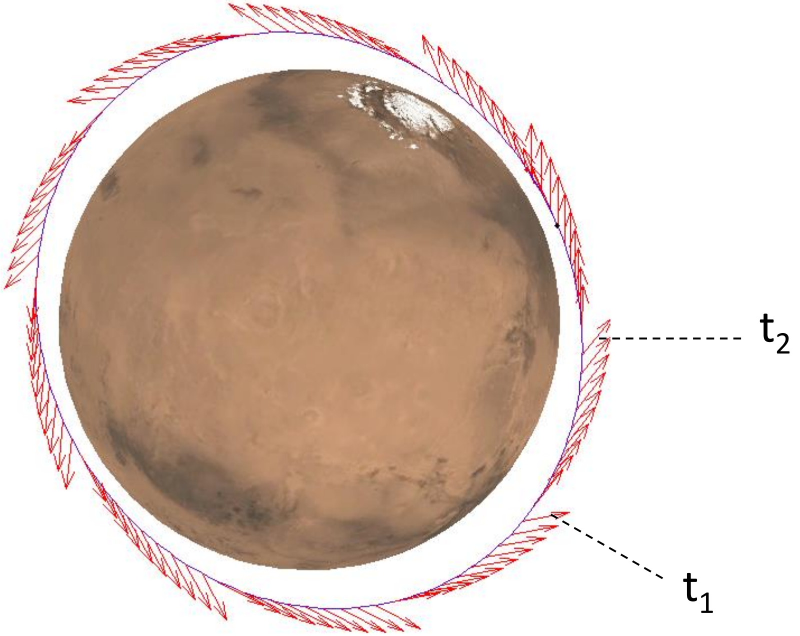
Stay ≥ 500 km from Mars

x(t): spacecraft state v(t): thrust vector w: parameters

The optimal Control problem:

Control Dynamics Limitation
(simplest example):

$$\frac{dv(t)}{dt} = 0 \qquad \forall\, t \in (t_1, t_2)$$



Dynamic limitations represent engineering or operational requirements. For example, continuous, very slow, slewing of the spacecraft to change the thrust direction is not always desirable. Instead, the thrust direction is altered quickly at regular intervals.

$x(t)$: spacecraft state $v(t)$: thrust vector w: parameters

The optimal Control problem:

Objective: $$maximize_{v(t),w}\ (Spacecraft\ final\ mass)$$

State equation: $$\frac{dx(t)}{dt} = T(x(t), v(t), w, t)$$

Physics of Space Flight

Initial Condition: $$x(t_0) = \Gamma(w)$$

Target Final State: $$\Psi(x(t_f), v(t_f), w, t_f) = \ or\ \leq k_1$$

Intermediate State: $$\Psi(x(t_i), v(t_i), w, t_i) = \ or\ \leq k_i$$

x(t): spacecraft state   v(t): thrust vector   w: parameters

What is in the state equation:

Spacecraft:

$$\frac{dx}{dt} = \begin{bmatrix} \overrightarrow{x}_{4:6}(t) \\ \frac{\overrightarrow{v}(t)}{x_7(t)} + \frac{\overrightarrow{Rad}(x,t)}{x_7(t)} + \sum_{i=1}^{N_{bodies}} \overrightarrow{Grav_i}(x,t) \\ \dot{m}(v,x,t) \end{bmatrix} \begin{array}{l} \leftarrow \text{Velocity} \\ \leftarrow \text{Acceleration} \\ \leftarrow \text{Mass change} \end{array}$$

x(t): spacecraft state v(t): thrust vector w: parameters

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Thrust: acceleration = thrust force/mass

x(t): spacecraft state   v(t): thrust vector   w: parameters

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$$\frac{dx}{dt} = \begin{bmatrix} \vec{x}_{4:6}(t) \\ \frac{\vec{v}(t)}{x_7(t)} + \frac{\vec{Rad}(x,t)}{x_7(t)} + \sum_{i=1}^{N_{bodies}} \vec{Grav}_i(x,t) \\ \dot{m}(v,x,t) \end{bmatrix}$$

Velocity

Acceleration

Mass change

Thrust

Radiation pressure: acceleration = radiation force/mass

x(t): spacecraft state v(t): thrust vector w: parameters

What is in the state equation:

Spacecraft:

$$\frac{dx}{dt} = \begin{bmatrix} \vec{x}_{4:6}(t) \\ \frac{\vec{v}(t)}{x_7(t)} + \frac{\vec{Rad}(x,t)}{x_7(t)} + \sum_{i=1}^{N_{bodies}} \vec{Grav}_i(x,t) \\ \dot{m}(v,x,t) \end{bmatrix} \begin{array}{l} \leftarrow \text{Velocity} \\ \leftarrow \text{Acceleration} \\ \leftarrow \text{Mass change} \end{array}$$

Thrust

Radiation pressure

Ion thruster propellant rate: dmass/dt = function of spacecraft thrust magnitude, spacecraft position, and **TIME**.

dmass/dt is a **discontinuous** function of time because we must change engines as we travel.

x(t): spacecraft state   v(t): thrust vector   w: parameters

What is in the state equation:

Spacecraft:

$$\frac{dx}{dt} = \begin{bmatrix} \vec{x}_{4:6}(t) \\ \frac{\vec{v}(t)}{x_7(t)} + \frac{\vec{Rad}(x,t)}{x_7(t)} + \sum_{i=1}^{N_{bodies}} \vec{Grav}_i(x,t) \\ \dot{m}(v,x,t) \end{bmatrix} \begin{array}{l} \leftarrow \text{Velocity} \\ \leftarrow \text{Acceleration} \\ \leftarrow \text{Mass change} \end{array}$$

Thrust

Radiation pressure

- Ion thruster propellant rate

Gravitational terms: N - body gravity, gravitational harmonics, and first order relativistic corrections

x(t): spacecraft state　　v(t): thrust vector　　w: parameters

What is in the state equation: Gravity

Simple point mass representation:

$$\vec{Grav_i} = -\frac{\mu_i r_i}{||r_i||^3}$$

Newton's Law of Gravity

Extended, rotating, body representation (spherical harmonics):

$$\vec{Grav_i} = [Q_i](t) \cdot \nabla \left\{ \frac{\mu_i}{r_i} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \frac{R_i^n}{r_i^n} P_{nm}(sin(\phi))(C_{nm} cos(m\lambda) + S_{nm} sin(m\lambda)) \right\}$$

Rotation matrix from inertial frame to body rotating frame

The optimal Control problem:

Objective: $$maximize_{v(t),w} \ (Spacecraft\ final\ mass)$$

State equation: $$\frac{dx(t)}{dt} = T(x(t), v(t), w, t)$$

Initial Condition: $$x(t_0) = \Gamma(w)$$

Launch Vehicle

Target Final State: $$\Psi(x(t_f), v(t_f), w, t_f) = \ or \ \leq k_1$$

Intermediate State: $$\Psi(x(t_i), v(t_i), w, t_i) = \ or \ \leq k_i$$

x(t): spacecraft state   v(t): thrust vector   w: parameters

An Example of the Initial Condition Equation:

$$\Gamma(w) = \begin{Bmatrix} initial\ position \\ initial\ velocity \\ initial\ mass \end{Bmatrix} = \begin{Bmatrix} \overrightarrow{X}\ (\Omega, \omega, \nu, C_3, R_p, i) \\ \overrightarrow{V}\ (\Omega, \omega, \nu, C_3, R_p, i) \\ mlv_c(c_3) + w_9 \end{Bmatrix}$$

An Example of the Initial Condition Equation:

Optimization parameters:
Launch vehicle final state

$$\Gamma(w) = \begin{Bmatrix} initial\ position \\ initial\ velocity \\ initial\ mass \end{Bmatrix} = \begin{Bmatrix} \overrightarrow{X}\left(\Omega, \omega, \nu, C_3, R_p, i\right) \\ \overrightarrow{V}\left(\Omega, \omega, \nu, C_3, R_p, i\right) \\ mlv_c(c_3) + w_9 \end{Bmatrix}$$

An Example of the Initial Condition Equation:

Position and velocity the Launch vehicle can deliver the Dawn spacecraft to immediately after the 3rd stage burnout

$$\Gamma(w) = \begin{Bmatrix} initial\ position \\ initial\ velocity \\ initial\ mass \end{Bmatrix} = \begin{Bmatrix} \vec{X}\left(\Omega, \omega, \nu, C_3, R_p, i\right) \\ \vec{V}\left(\Omega, \omega, \nu, C_3, R_p, i\right) \\ mlv_c(c_3) + w_9 \end{Bmatrix}$$

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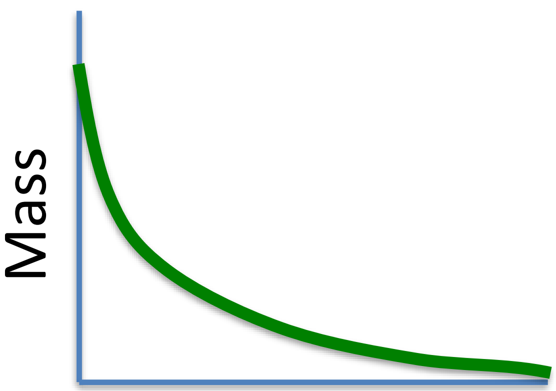


Launch vehicle performance:
Delivered mass versus delivered energy to the Earth Escape hyperbola

Navigating With Ion Engines …

- Many of the procedures and algorithms developed to guide traditional chemical propelled spacecraft through the Solar System **do not** extend to ion propelled spacecraft.
- Chemical engines are typically on for minutes and off (coasting) for years
- Ion engines are on for years (Dawn will likely operate its thrusters about 6 years!)
- Mission and trajectory design are much more difficult because of the near-continuous thruster operation.

Trajectory Design as an Optimization Problem

- Trajectory design is generally posed as an optimal control problem with a variety of (sometimes peculiar) constraints.

*Get from point **A** to point **B** delivering the **maximum payload***

Subject to the laws of physics, engineering constraints, and programmatic constraints

Models Required:

- Solar array performance as a function of temperature and illumination.
- Solar array degradation due to radiation damage
- Spacecraft subsystems (non-ion engine) power consumption
- Ion engine performance (generally non-linear!)
- Launch vehicle performance curves
- Mass distribution models for all gravitating bodies
- Spacecraft component reflectivity for radiation pressure
- Spacecraft attitude control system propellant usage

Constraints:

- Ion engine operational limits
- Launch vehicle ascent geometry limits
- Start, end, and total time of flight
- Propellant consumption (maximum tank size)
- Solar array thermal limits
- Ion engine thrust beam Sun relative direction constraints
- Periodic forced coasting for communications
- Trajectory failsafe constraint (robust against thrust outages)
- Targeted intermediate state conditions
- Targeted final state conditions

Characteristics of the OCP:

- Nonlinear objective, constraints, and state equation
- Non-convex
- System response is "Knife edged" (need 2nd order method)
- Control is discontinuous in time
- State equation is discontinuous in time
- State equation is non-autonomous
- Large changes in physical scale occur over the problem's time horizon
- Very high precision solutions are required for flight

Tough problem!

Method of Solution:

- Nonlinear optimal control based on Bellman's principal of optimality (*Bellman, 1957*), or more specifically the *Hamilton, Jacobi, Bellman* equation

- I developed an algorithm specifically to solve these types of problems:

"Static Dynamic Control" (*Whiffen, 1999)*

Vesta Trajectory Zoom

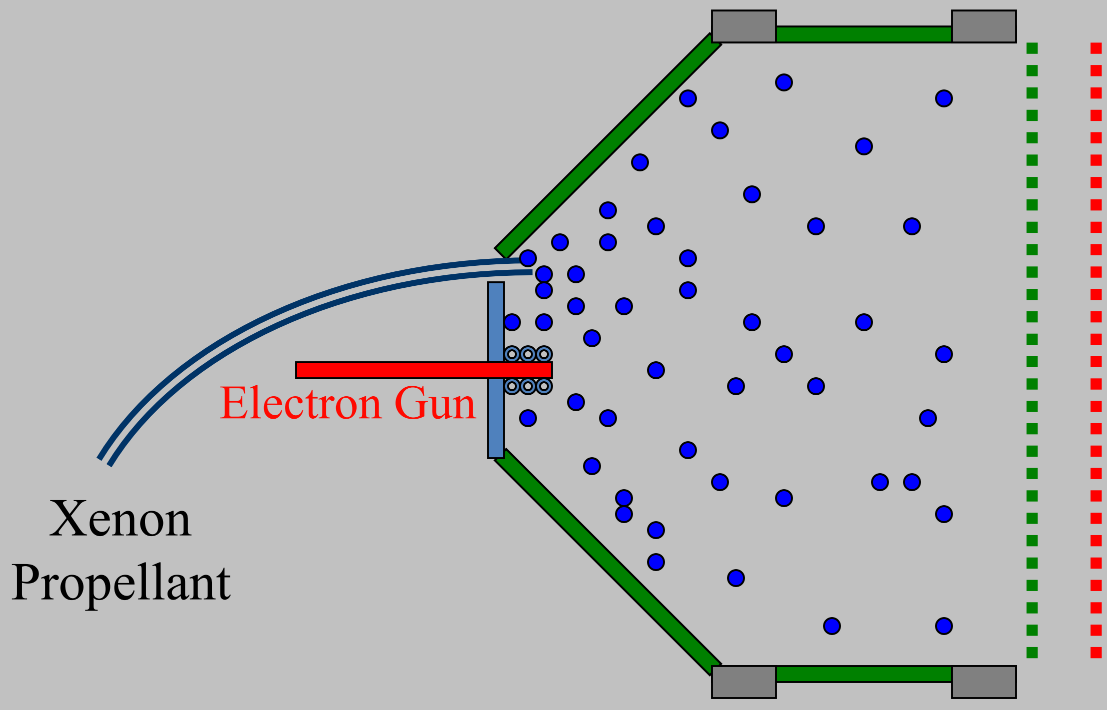
What Is An Ion Engine?

- Traditional chemical rockets are near the peak theoretical capability

- To get the propellant exiting much faster, we need a non-thermal means of propellant acceleration. Best chemical exhaust speed is about 13,000 [km/hr]

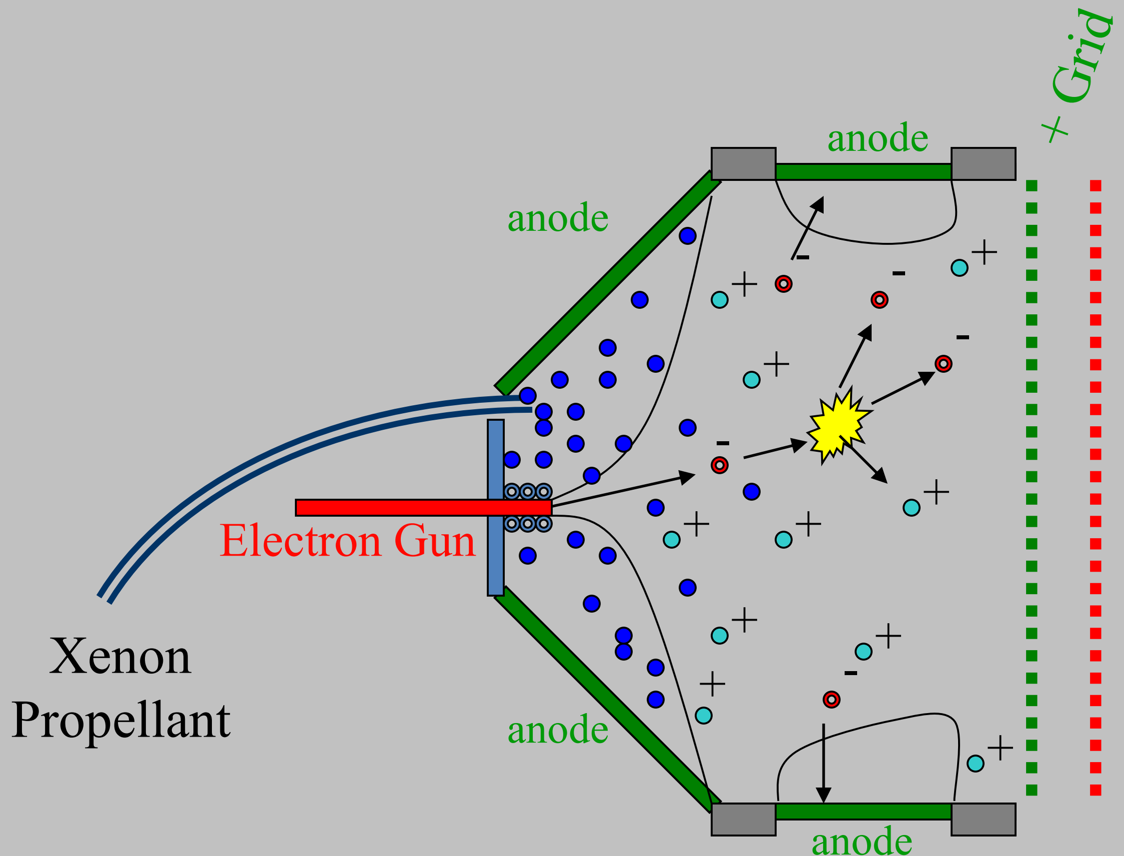
  The ion engine is really a particle accelerator. Particle accelerators on Earth are limited only by the speed of light.

How an Ion Engine Works



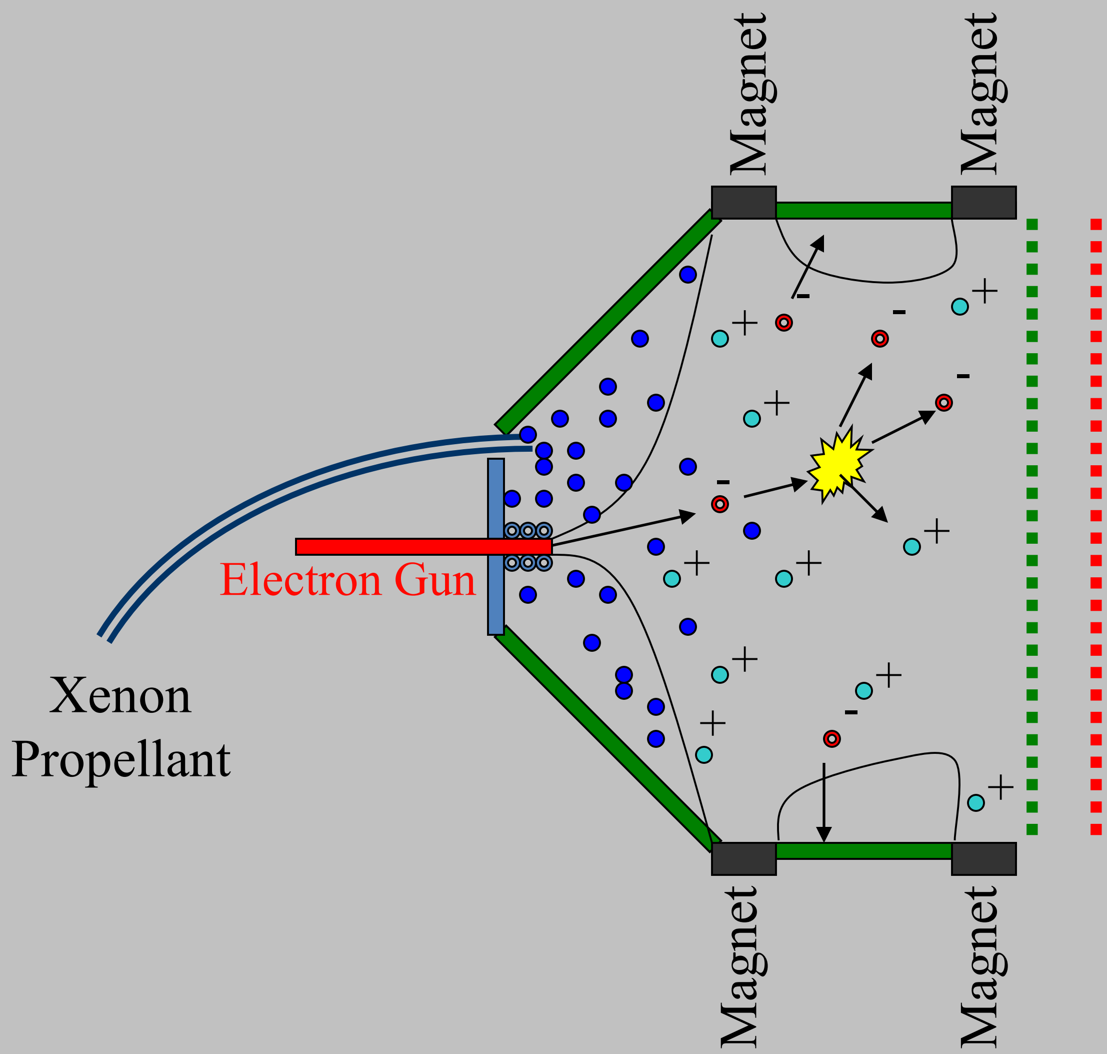
Xenon propellant is injected into the propulsion chamber

How an Ion Engine Works



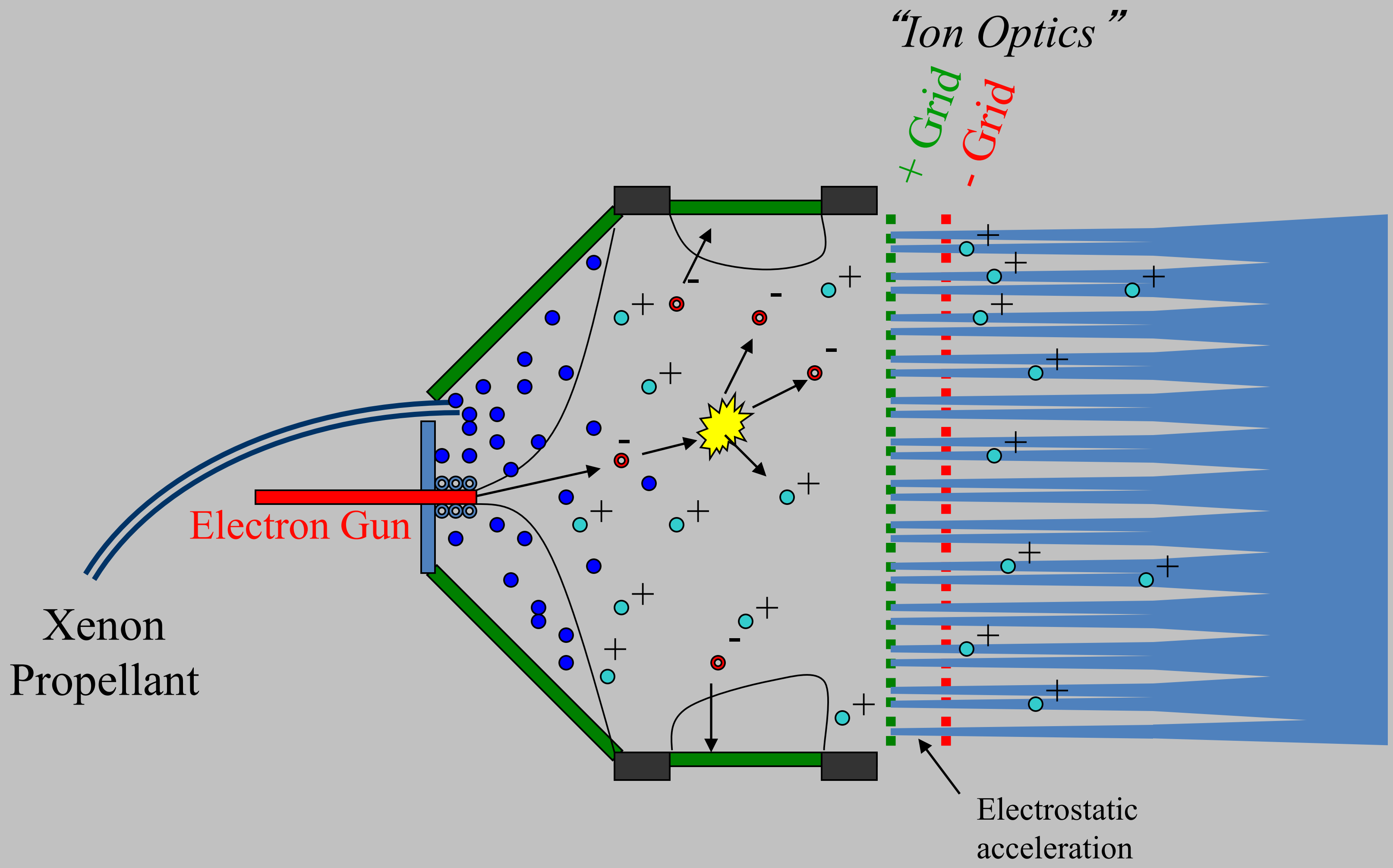
The propellant is ionized by electron bombardment, the + grid and thruster walls (anodes) absorb all the excess electrons.

How an Ion Engine Works



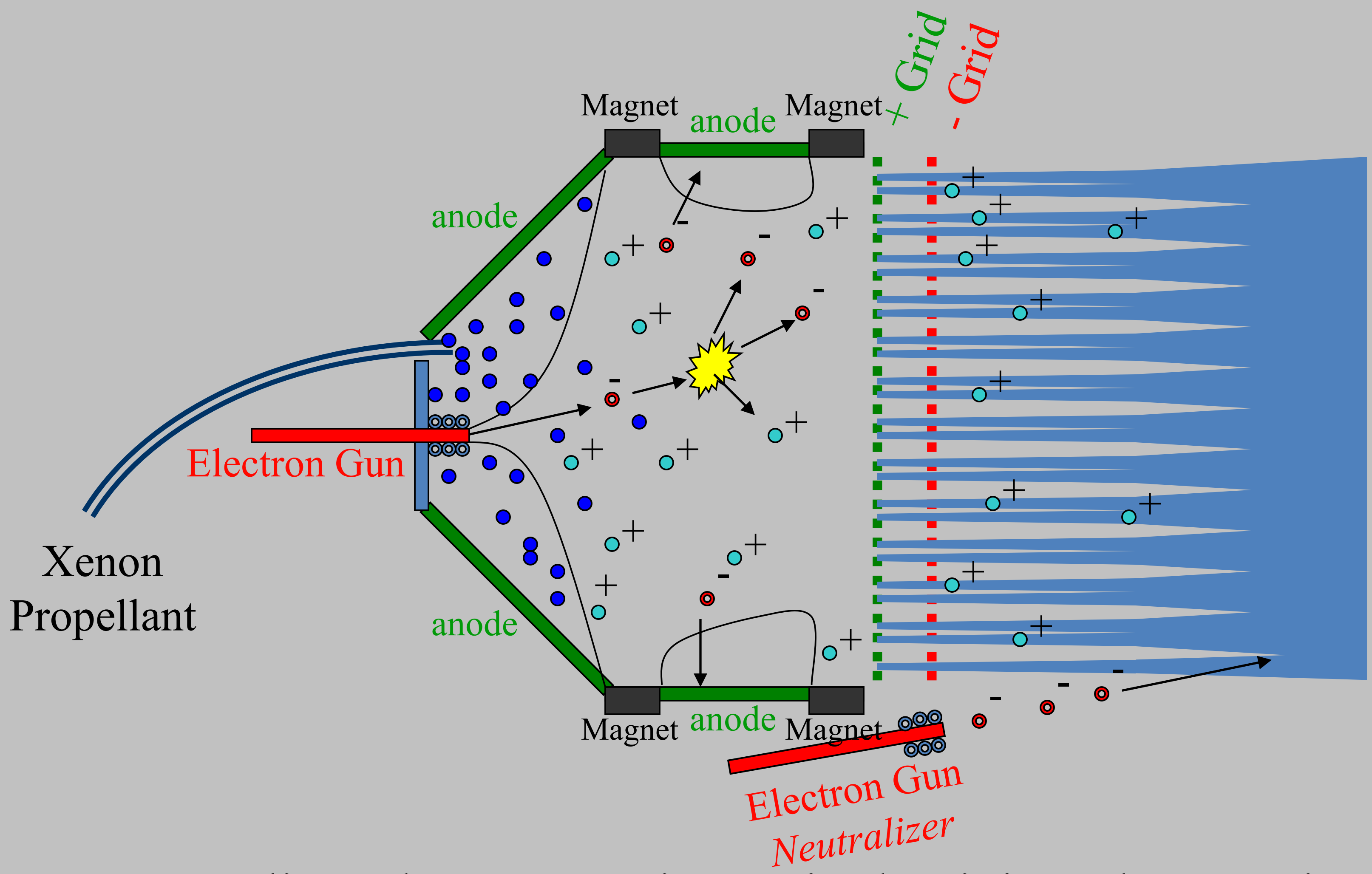
Permanent Ring magnets increase the electron residence time to improve ionization efficiency

How an Ion Engine Works

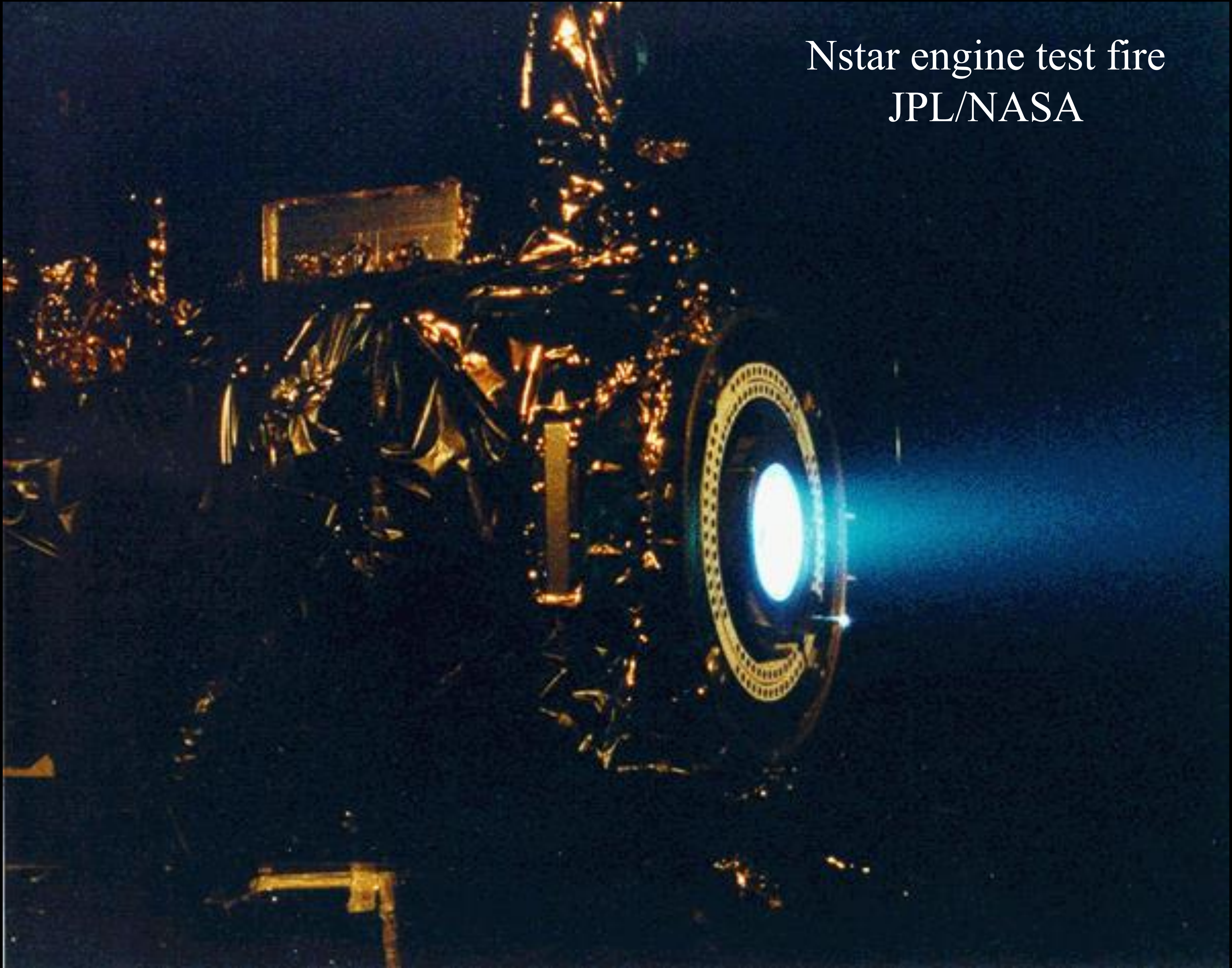


The ions diffuse towards the holes in the + grid, feel the - grid, are electrostatically accelerated to high speed, and focused through the holes on the - grid into space.

How an Ion Engine Works

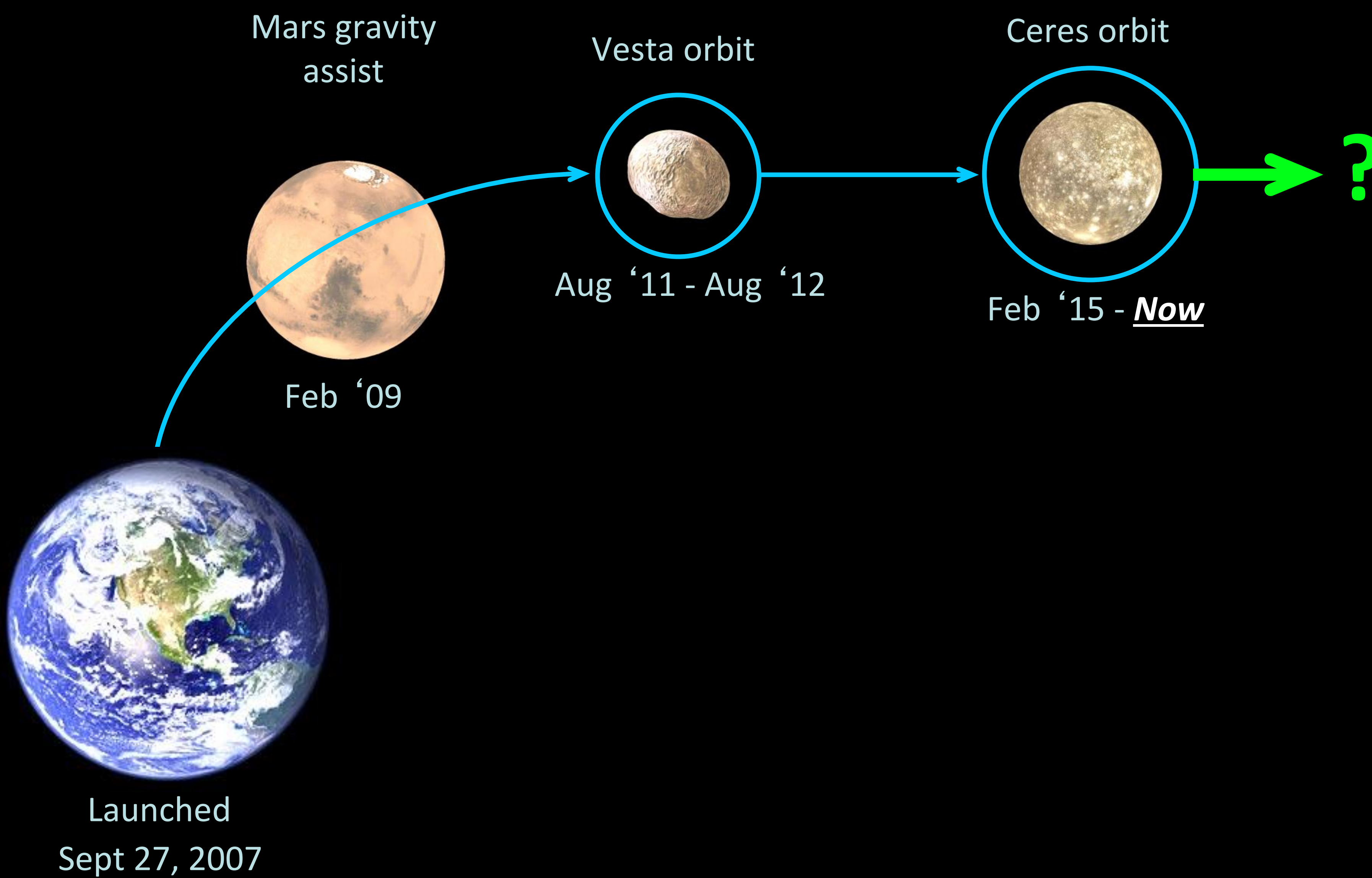


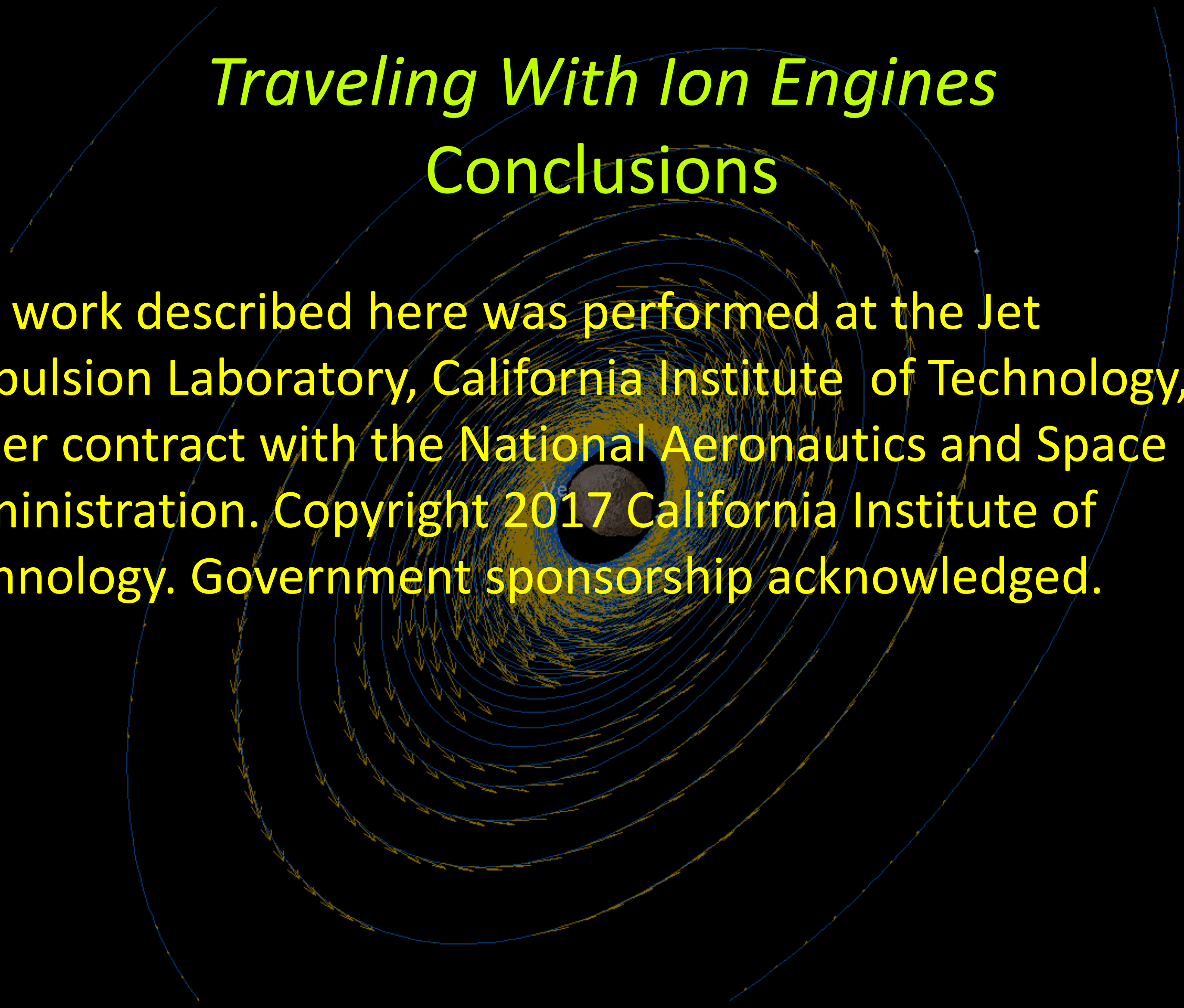
A neutralizer electron gun is required to inject electrons into The ion beam to keep the spacecraft from building up charge



Nstar engine test fire
JPL/NASA

Dawn Mission Itinerary

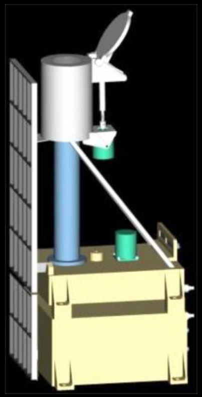




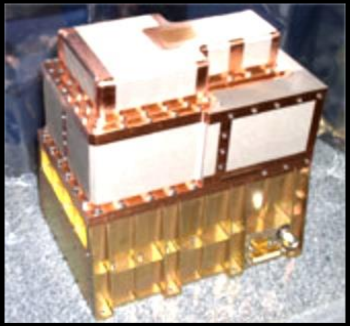
Traveling With Ion Engines
Conclusions

The work described here was performed at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration. Government sponsorship acknowledged.

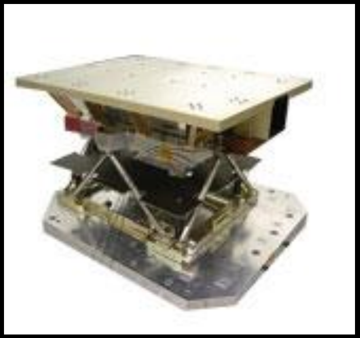
Scientific Instruments and objectives



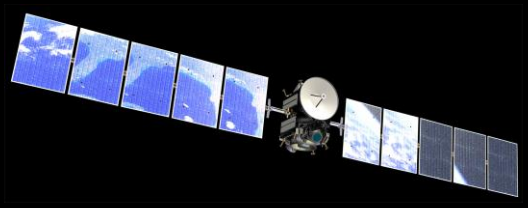
Framing Cameras



Gamma ray and Neutron Detector



Visible and Infrared spectrometer

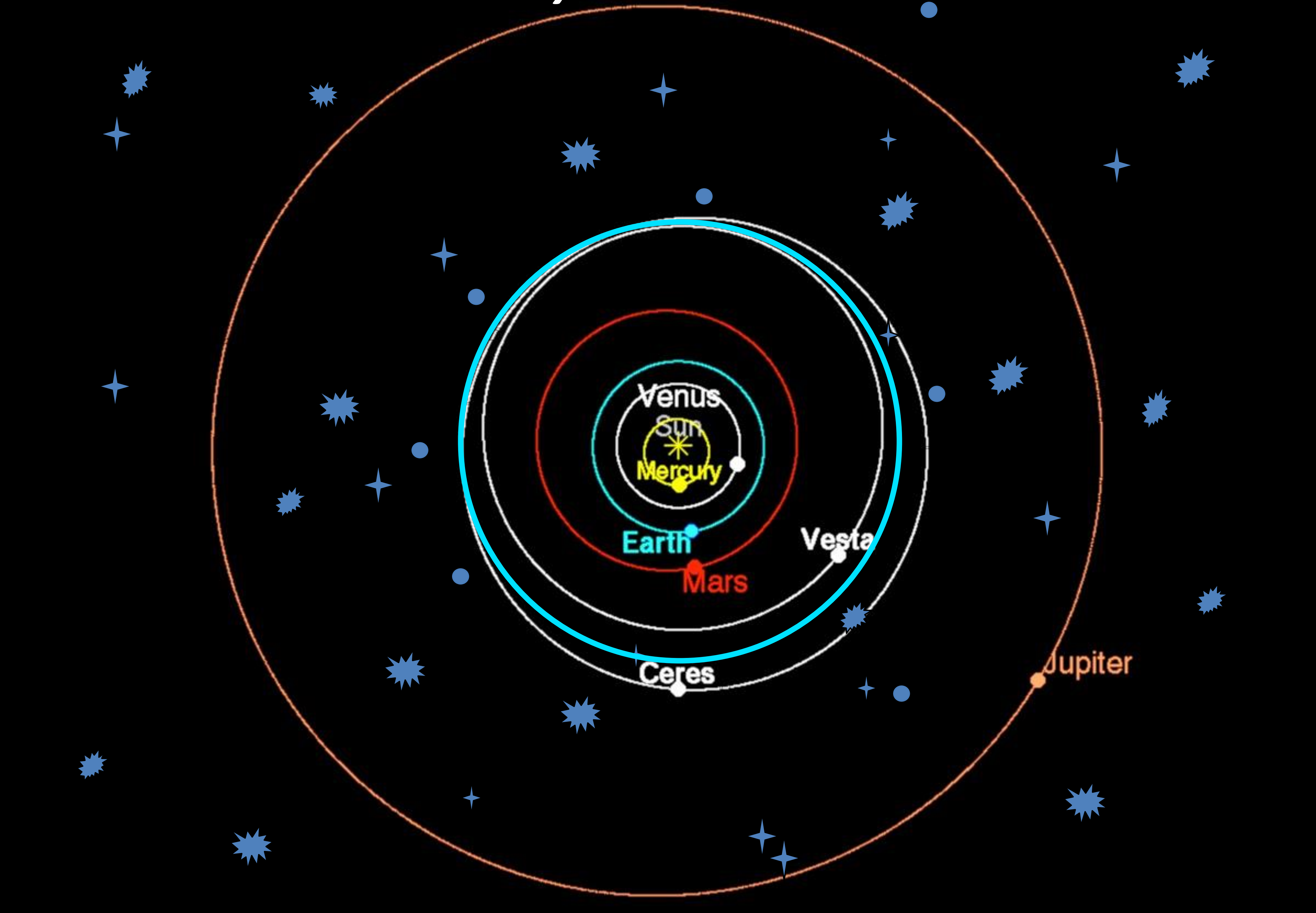


Gravity

At Both Vesta and Ceres:

- Global color map
- Topographic map
- Map elemental composition
- Map mineralogical composition
- Map gravity field
- Search for moons

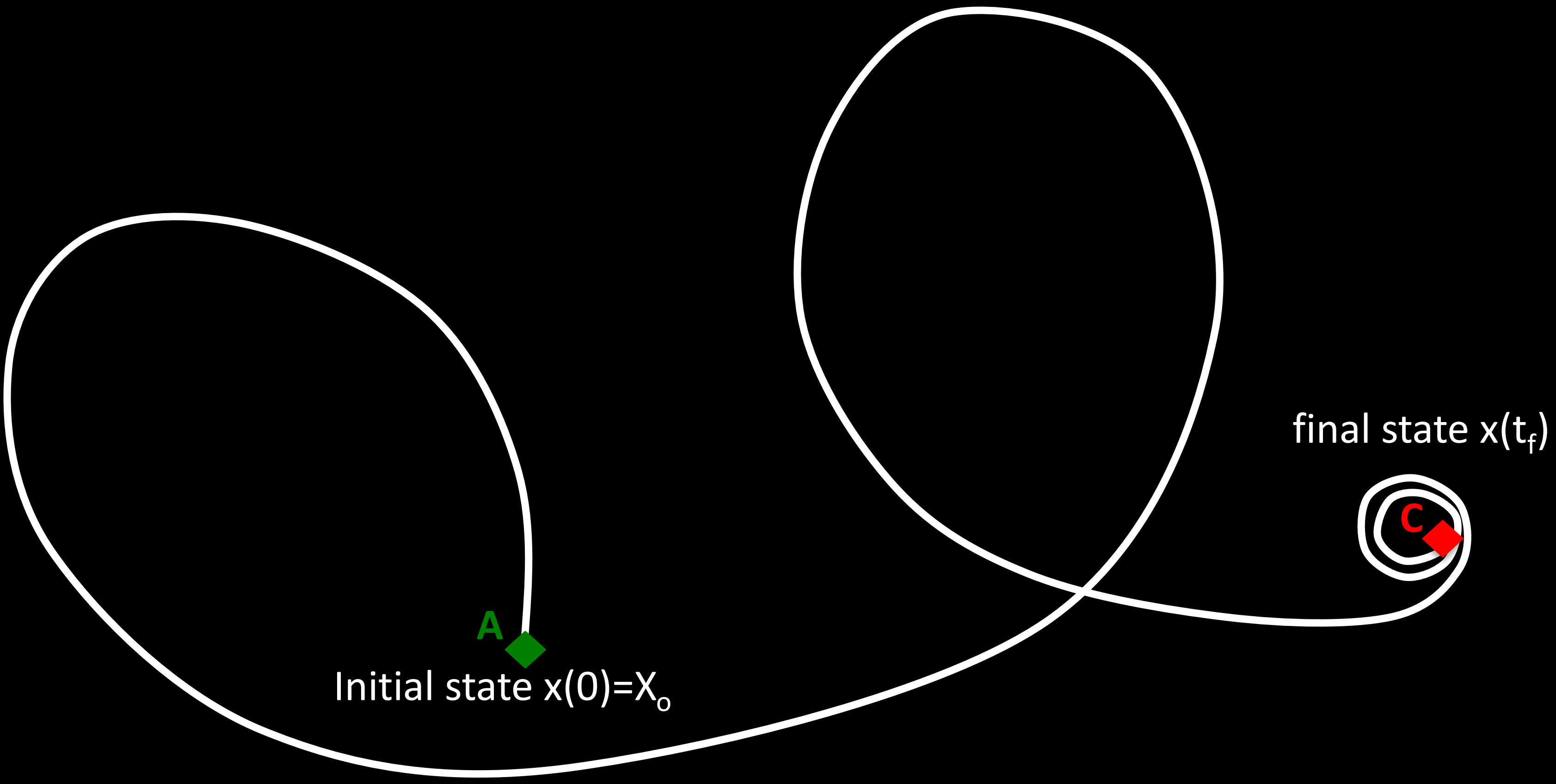
The Solar System "snow line"



Bellman's Principal

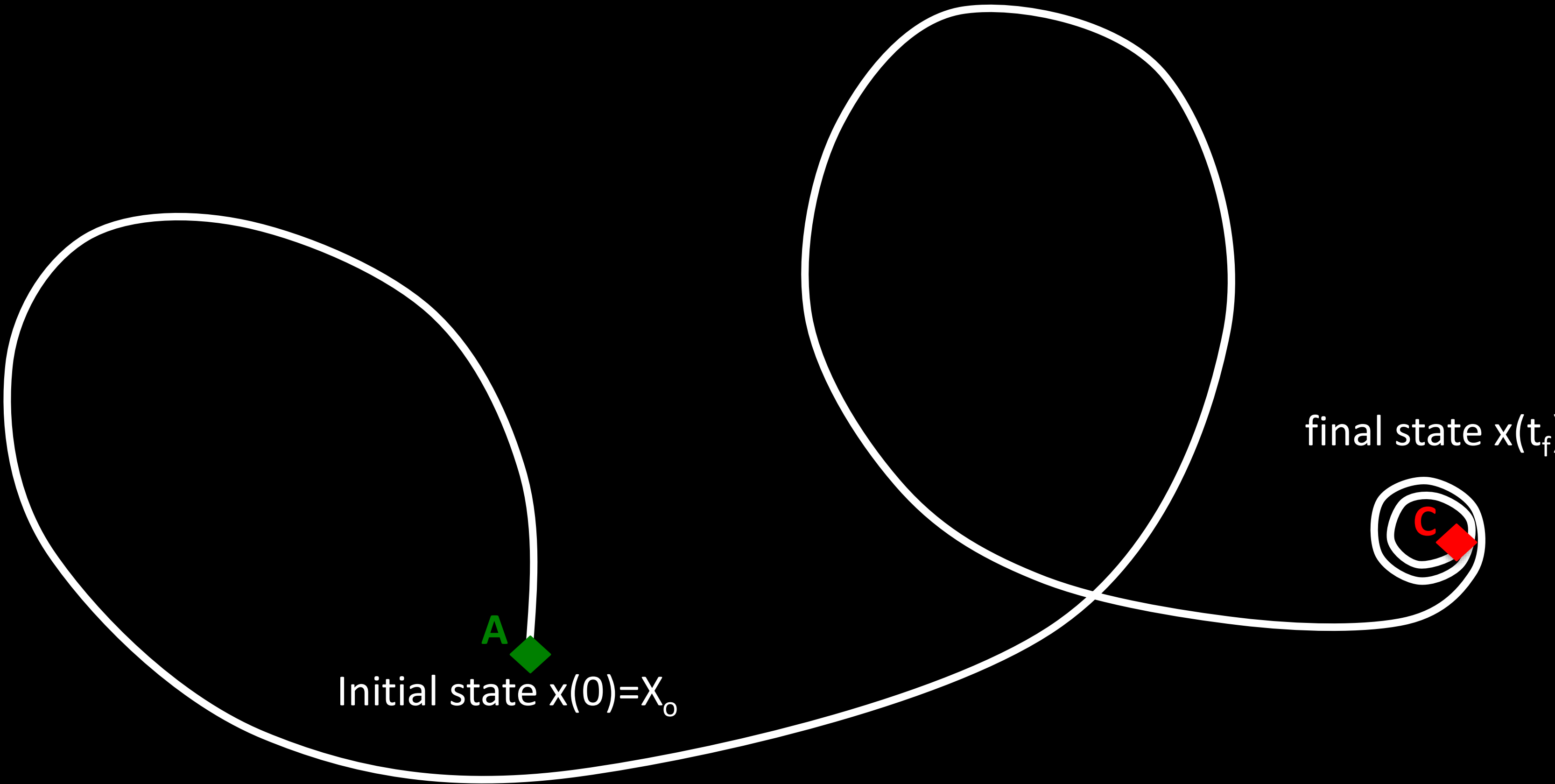
An optimal trajectory has the property that whatever the initial state and the initial control were, the remaining control must constitute an optimal trajectory with regard to the state resulting from the initial controls.

Bellman's Principal



Suppose this is an optimal trajectory from points A to C

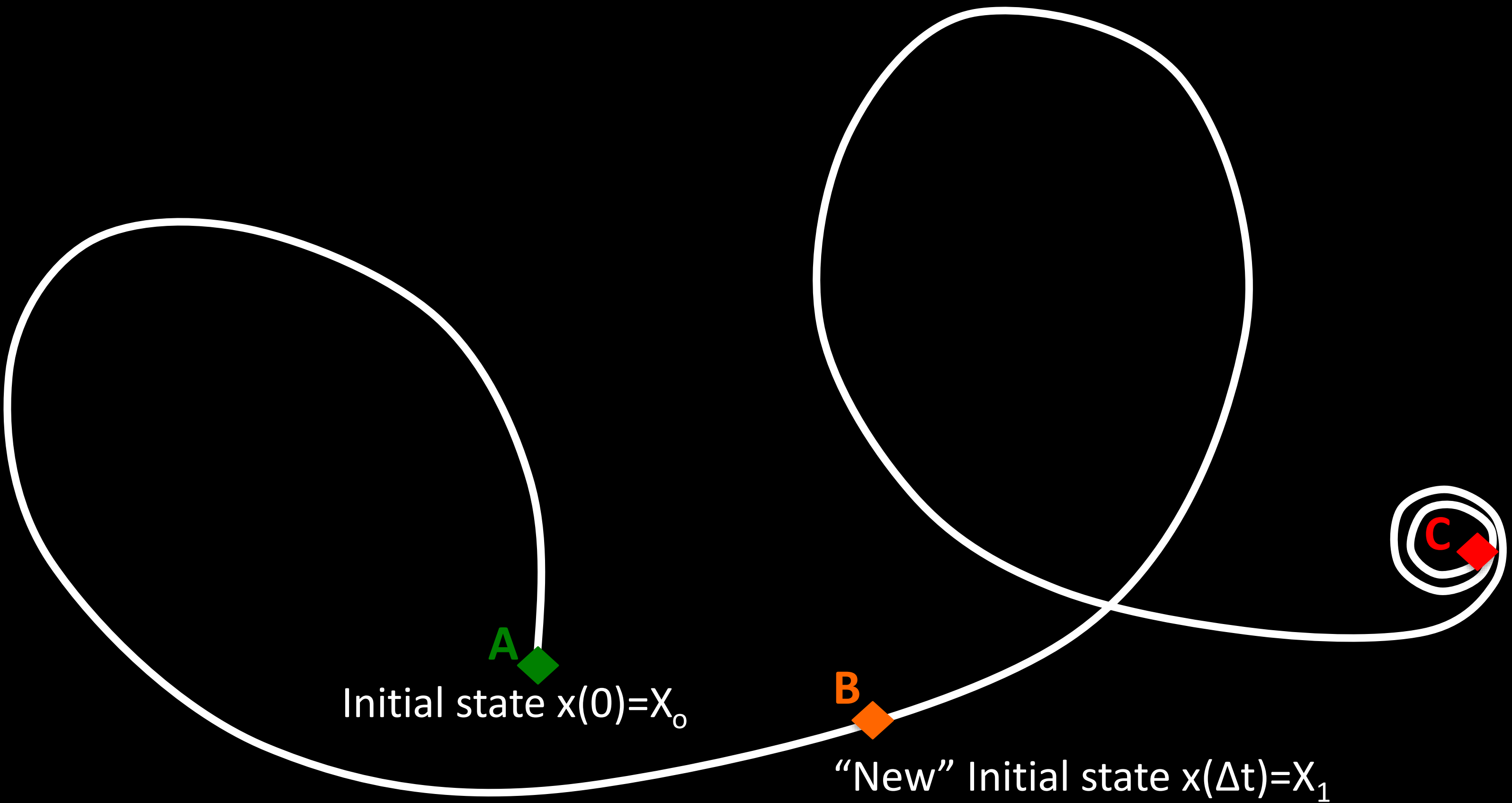
Bellman's Principal



The Optimal control **$u^*(t)$** $t=0$ to t_f delivers us from **A** to **C**

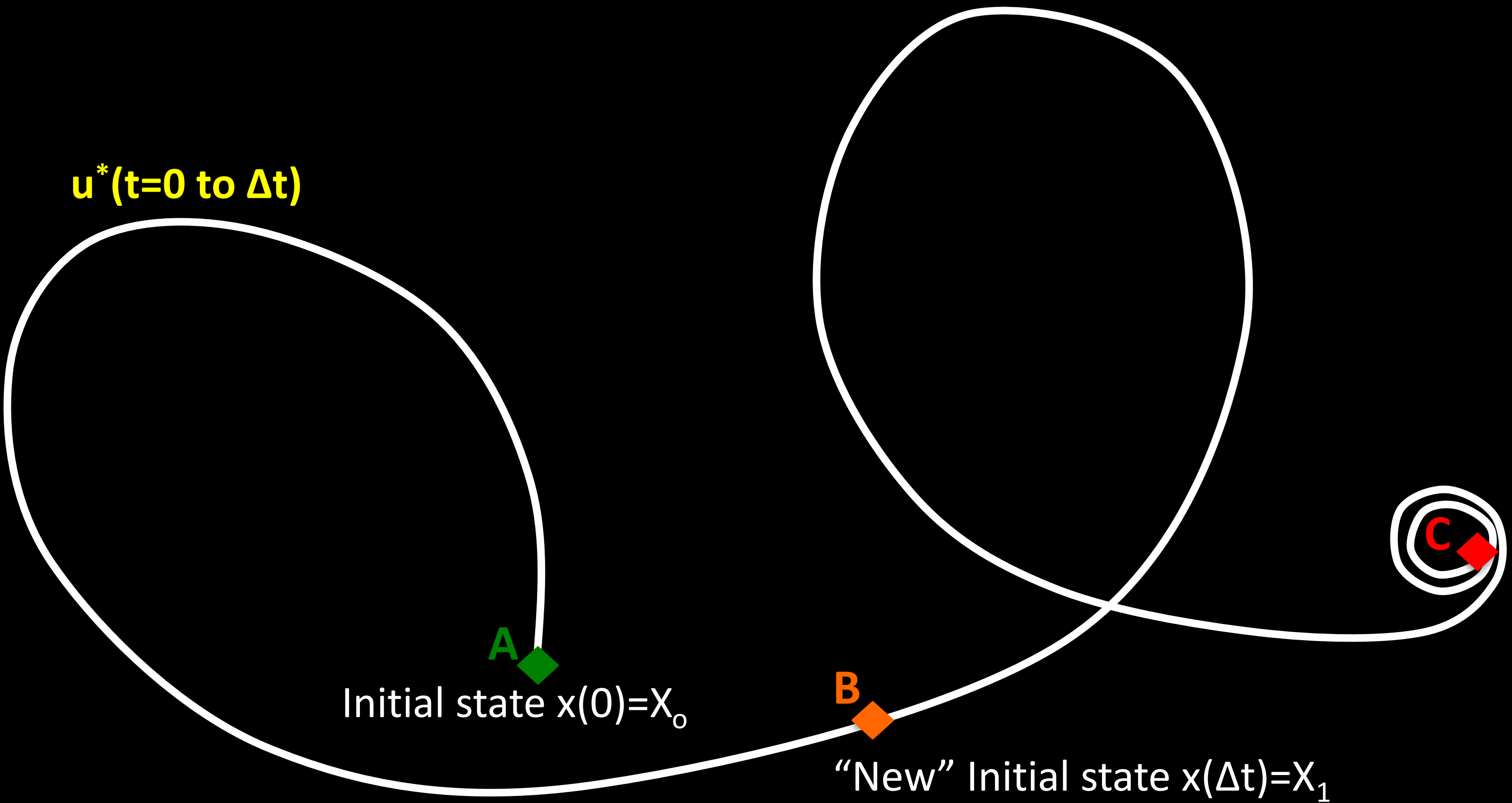
AND: minimizes $\int_{t=0}^{t_f} F(x,u,t)\,dt$ *subject to* $dx/dt = T(x(t),u(t),t)$

Bellman's Principal



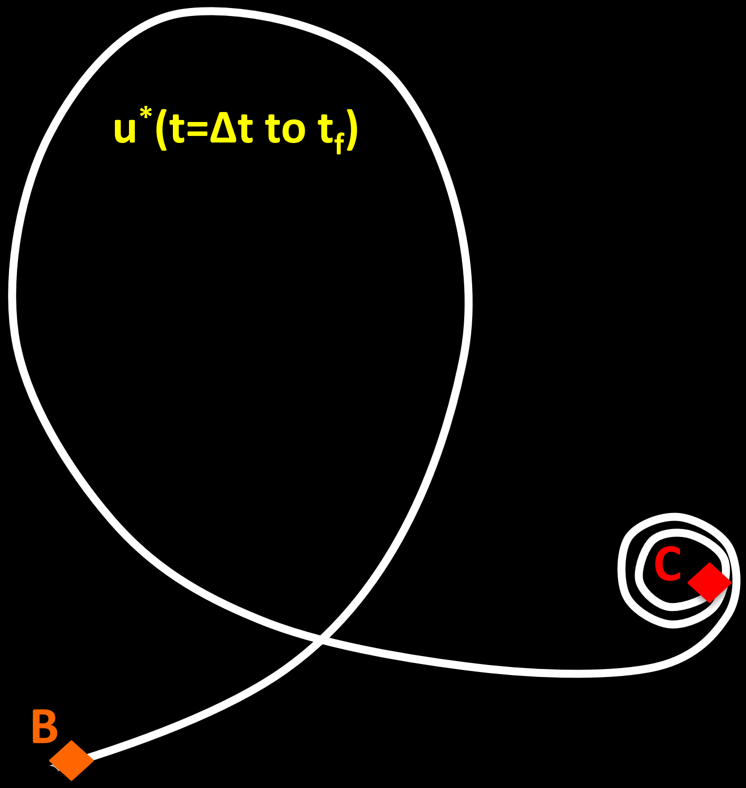
Consider an intermediate point B

Bellman's Principal



The optimal control $u^*(t=0 \text{ to } \Delta t)$ got us to point **B**

Bellman's Principal



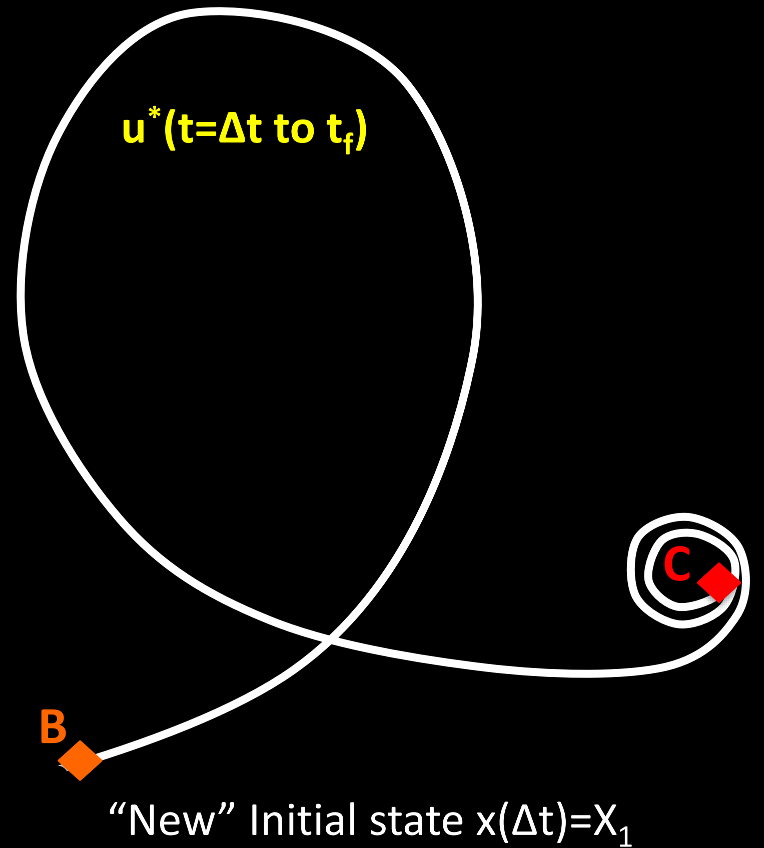
"New" Initial state $x(\Delta t)=X_1$

Now forget how we got to **B**, consider the optimal control problem from **B** to **C** with the same performance measure

Bellman's Principal

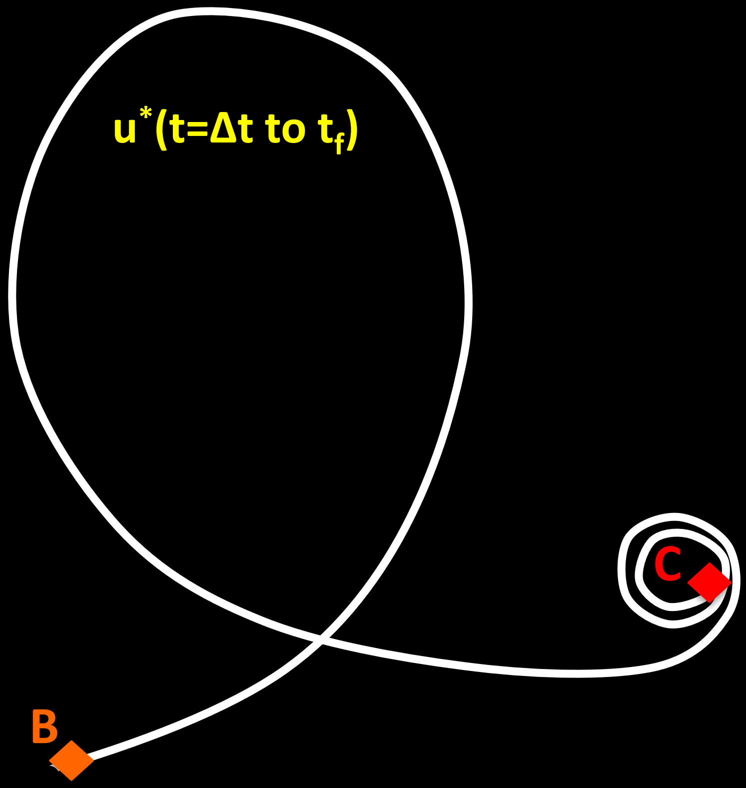
Bellman says the answer is the remaining optimal control you obtained for the full A to C problem: **$u^*(t=\Delta t \text{ to } t_f)$**

$$\int_{t=\Delta t}^{t_f} F(x,\mathbf{u}^*,t)\,dt$$



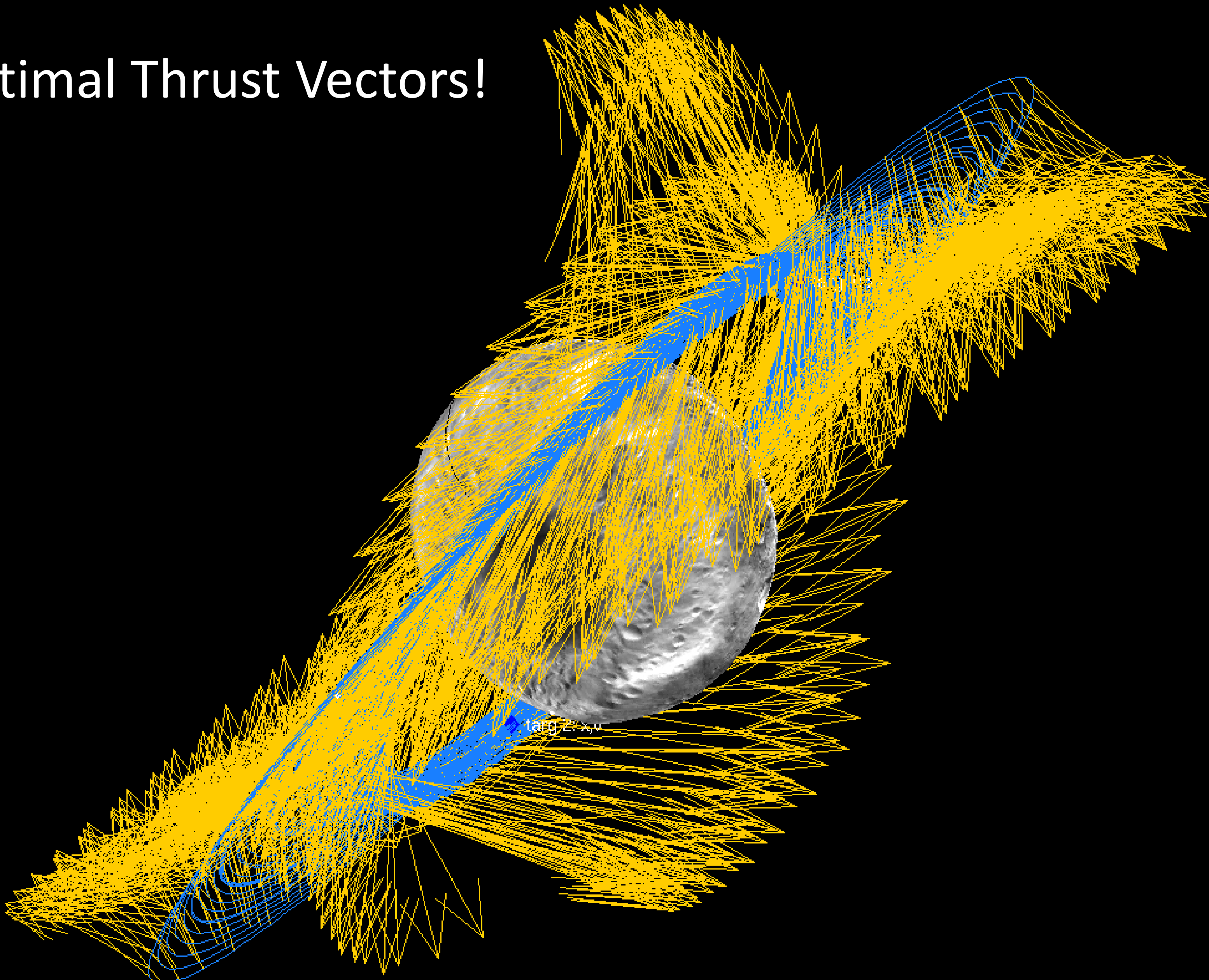
Bellman's Principal

Proof by contradiction: assume a better control exists B to C, then the original A to C problem must not be optimal – it can be improved by using the better B to C solution.



"New" Initial state $x(\Delta t)=X_1$

Optimal Thrust Vectors!



Traveling With Ion Engines

Conclusions

- The great improvement in propulsive efficiency enables previously impossible missions like Dawn
- Despite this, ion propulsion remains under utilized
- Trajectory design for ion propelled spacecraft is significantly more difficult than traditional problems - and remains a very active area of research.
- The "SDC" algorithm based on Bellman's Principal has been demonstrated to be more effective at solving ion engine trajectory problems than any other technique.
- *Watch the news for Dawn's extended mission this summer!*

Dawn at Ceres

[illegible]

Maneuverability with ion propulsion

Static Dynamic Control

- General Form of the objective:

$$J^* = \min_{w,v(t)} \int_{t_0}^{t_N} F(x(t), v(t), w, t)dt + \sum_{i=1}^{N} G(x(t_i), v(t_i), w, t_i, i)$$

- The general state equation:

$$\frac{dx(t)}{dt} = T(x(t), v(t), w, t) \qquad x(t = t_0) = \Gamma(w)$$

Static Dynamic Control

- General Form of the objective:

$$J^* = \min_{w,v(t)} \int_{t_0}^{t_N} F(x(t), v(t), w, t)dt + \sum_{i=1}^{N} G(x(t_i), v(t_i), w, t_i, i)$$

- The general state equation:

$$\frac{dx(t)}{dt} = T(x(t), v(t), w, t) \qquad x(t = t_0) = \Gamma(w)$$

- Dynamic limitations on the control (Optional)

$$v(t) = \left| \begin{array}{ll} f(u_1, w, t, 1) & for \;\; t = t_0 \;\; to \;\; t_1 \\ f(u_2, w, t, 2) & for \;\; t = t_1 \;\; to \;\; t_2 \\ \vdots & \vdots \\ f(u_N, w, t, N) & for \;\; t = t_{N-1} \;\; to \;\; t_N. \end{array} \right|$$

"*Period 1*"
"*Period 2*"
"*Period N*"

Static Dynamic Control (Period Formulation)

- Define the (not necessarily optimal) objective going forward for period ***N***:

$$J(x,u_N,w,t) \doteq \int_t^{t_N} F(x(\tau),f(u_N,w,\tau,N),w,\tau)d\tau + G(x(t_N),u_N,w,t_N,N)$$

- Goal: Develop a system of O.D.E.s that generates the derivatives of ***J*** with respect to ***x, u,*** and ***w.*** Next, use those derivatives at $\boldsymbol{t_{N-1}}$ make a locally optimal feedback law for ***u*** in covering **period N**.

- By analogy, construct a feedback law for the **period N-1** assuming the feedback law for period **N** is used.

- Repeat this process backward to **period 1**. Use the derivatives of ***J*** with respect to ***w*** at $\boldsymbol{t_o}$ to compute an update for ***w***.

Static Dynamic Control

Definition of ***J***:

$$J(x, u_N, w, t) \doteq \int_t^{t_N} F(x(\tau), f(u_N, w, \tau, N), w, \tau) d\tau + G(x(t_N), u_N, w, t_N, N)$$

we can write ***J*** at time ***t*** as a function of ***J*** at time ***t+Δt*** by splitting the integral:

$$J(x, u_N, w, t) = \int_t^{t+\Delta t} F(x(\tau), f(u_N, w, \tau, N), w, \tau) d\tau + J(x(t+\Delta t), u_N, w, t+\Delta t)$$

Static Dynamic Control

Definition of ***J***:

$$J(x, u_N, w, t) \doteq \int_t^{t_N} F(x(\tau), f(u_N, w, \tau, N), w, \tau) d\tau + G(x(t_N), u_N, w, t_N, N)$$

we can write ***J*** at time ***t*** as a function of ***J*** at time ***t+Δt*** by splitting the integral:

$$J(x, u_N, w, t) = \int_t^{t+\Delta t} F(x(\tau), f(u_N, w, \tau, N), w, \tau) d\tau + J(x(t+\Delta t), u_N, w, t+\Delta t)$$

Taylor series

$$J(x(t+\Delta t), u_N, w, t+\Delta t) = J(x(t), u_N, w, t) + (J_t + J_x^t\, \dot{x} + J_u^t\, \dot{u}_N + J_w^t\, \dot{w})\Delta t + O(\Delta t^2)$$

Static Dynamic Control

Definition of ***J***:

$$J(x,u_N,w,t) \doteq \int_t^{t_N} F(x(\tau), f(u_N,w,\tau,N), w, \tau)d\tau + G(x(t_N), u_N, w, t_N, N)$$

we can write ***J*** at time ***t*** as a function of ***J*** at time ***t+Δt*** by splitting the integral:

$$J(x,u_N,w,t) = \int_t^{t+\Delta t} F(x(\tau), f(u_N,w,\tau,N), w, \tau)d\tau + J(x(t+\Delta t), u_N, w, t+\Delta t)$$

Taylor series

$$J(x(t+\Delta t), u_N, w, t+\Delta t) = J(x(t), u_N, w, t) + (J_t + J_x^t\, \dot{x} + J_u^t\, \cancel{\dot{u}_N} + J_w^t\, \cancel{\dot{w}})\Delta t + O(\Delta t^2)$$

Remove terms that are zero

$$J(x(t+\Delta t), u_N, w, t+\Delta t) = J(x(t), u_N, w, t) + J_t \Delta t + J_x^t T \Delta t + O(\Delta t^2)$$

Static Dynamic Control

Substituting the Taylor series back in:

$$-J_t\Delta t = \int_t^{t+\Delta t} F(x(\tau), f(u_N, w, \tau, N), w, \tau)d\tau + J_x^t T\Delta t + O(\Delta t^2)$$

Next divide by ***Δ t*** and let ***Δ t*** go to zero:

$$-J_t(x, u_N, w, t) = F(x, f(u_N, w, t, N), w, t) + J_x^t(x, u_N, w, t)T(x, f(u_N, w, t, N), w, t)$$

This is a partial differential equation for ***J***. It can be differentiated to obtain analogous equations for the first two derivatives of ***J*** with respect to ***x, u,*** and ***w.***

Static Dynamic Control

P.D.Es for the first two derivatives:

$$-J_{tx} = F_x + J_{xx}T + T_x^t J_x$$

$$-J_{tu} = F_u + J_{xu}^t T + T_u^t J_x$$

$$-J_{tw} = F_w + J_{xw}^t T + T_w^t J_x$$

Subscripts denote derivatives

An example of one of the six P.D.E.s for the second derivatives

$$-J_{txx} = F_{xx} + \sum_{i=1}^{n} J_{xxx}[:,:,i]T[i] + J_{xx}T_x + T_x^t J_{xx} + \sum_{i=1}^{n} J_x[i]T_{xx}[i,:,:]$$

Static Dynamic Control

P.D.Es for the first two derivatives:

$$-J_{tx} = F_x + J_{xx}T + T_x^t J_x$$

$$-J_{tu} = F_u + J_{xu}^t T + T_u^t J_x$$

$$-J_{tw} = F_w + J_{xw}^t T + T_w^t J_x$$

An example of one of the six P.D.E.s for the second derivatives

$$-J_{txx} = F_{xx} + \sum_{i=1}^{n} J_{xxx}[:, :, i]T[i] + J_{xx}T_x + T_x^t J_{xx} + \sum_{i=1}^{n} J_x[i]T_{xx}[i, :, :]$$

Ugh!

Static Dynamic Control

We can develop a system of <u>ordinary differential equations</u> to find ***J*** and its derivatives by using the definition of the total time derivative. ***J*** is both an *explicit* function of time and an *implicit* function of time through the state ***x*** time evolution:

$$\dot{J} = J_t + J_x^t T$$

Where: $$T = \frac{\partial x}{\partial t}$$

Static Dynamic Control

We can develop a system of <u>ordinary differential equations</u> to find ***J*** and its derivatives by using the definition of the total time derivative. ***J*** is both an *explicit* function of time and an *implicit* function of time through the state ***x*** time evolution:

$$\dot{J} = J_t + J_x^t T$$

Similarly for the derivatives of ***J***:

$$\dot{J}_x = J_{xt} + J_{xx} T$$

$$\dot{J}_u = J_{ut} + J_{xu}^t T$$

Etc.

Static Dynamic Control

Substituting the total time derivative back into the original P.D.E.s we get O.D.E.s for the derivatives of ***J***:

$$\dot{J} = -F$$

$$\dot{J}_x = -F_x - T_x^t J_x$$

$$\dot{J}_u = -F_u - T_u^t J_x$$

$$J_w = -F_w - T_w^t J_x$$

Example of 1 of the 6 second order equations:

$$\dot{J}_{xu} = -F_{xu} - J_{xx} T_u - T_x^t J_{xu} - \sum_{i=1}^{n} J_x[i] T_{xu}[i, :, :]$$

Static Dynamic Control

Substituting the total time derivative back into the original P.D.E.s we get O.D.E.s for the derivatives of ***J***:

$$\dot{J} = -F$$

$$\dot{J}_x = -F_x - T_x^t J_x$$

$$\dot{J}_u = -F_u - T_u^t J_x$$

$$J_w = -F_w - T_w^t J_x$$

Example of 1 of the 6 second order equations:

$$\dot{J}_{xu} = -F_{xu} - J_{xx} T_u - T_x^t J_{xu} - \sum_{i=1}^{n} J_x[i] T_{xu}[i, :, :]$$

Notice that there is no longer a third derivative of J present – Yay!

Static Dynamic Control

The terminal condition for each O.D.E. is the corresponding derivative of the terminal cost function ***G***:

$$J(x(t_N), u_N, w, t_N) = G(x(t_N), u_N, w, t_N, N)$$

$$J_x(x(t_N), u_N, w, t_N) = G_x(x(t_N), u_N, w, t_N, N)$$

*And so on for all first and second derivatives of **J**…*

Static Dynamic Control

The first and second derivatives of ***J*** at time t_{N-1} can be obtained by integrating the systems of O.D.E.s backward in time from t_N to t_{N-1}.

Given the first and second derivatives of ***J*** at time t_{N-1} (denoted $\overline{J}$) a Taylor series expansion of ***J*** at time t_{N-1} is:

$$\begin{aligned}\hat{J}(\delta x, \delta u_N, \delta w) \ \doteq \ & \overline{J} + \overline{J}_x^t \delta x + \overline{J}_u^t \delta u_N + \overline{J}_w^t \delta w + \frac{1}{2}\delta x^t \overline{J}_{xx} \delta x + \delta x^t \overline{J}_{xu} \delta u_N + \delta w^t \overline{J}_{wu} \delta u_N \\ & + \frac{1}{2}\delta u_N^t \overline{J}_{uu} \delta u_N + \frac{1}{2}\delta w^t \overline{J}_{ww} \delta w + \delta x^t \overline{J}_{xw} \delta w.\end{aligned}$$

Static Dynamic Control

The first and second derivatives of ***J*** at time $\boldsymbol{t_{N-1}}$ can be obtained by integrating the systems of O.D.E.s backward in time from $\boldsymbol{t_N}$ to $\boldsymbol{t_{N-1}}$.

Given the first and second derivatives of ***J*** at time $\boldsymbol{t_{N-1}}$ (denoted $\overline{J}$) a Taylor series expansion of ***J*** at time $\boldsymbol{t_{N-1}}$ is:

$$\begin{aligned}\hat{J}(\delta x, \delta u_N, \delta w) \doteq\; & \overline{J} + \overline{J}_x^t \delta x + \overline{J}_u^t \delta u_N + \overline{J}_w^t \delta w + \frac{1}{2}\delta x^t \overline{J}_{xx}\delta x + \delta x^t \overline{J}_{xu}\delta u_N + \delta w^t \overline{J}_{wu}\delta u_N \\ & + \frac{1}{2}\delta u_N^t \overline{J}_{uu}\delta u_N + \frac{1}{2}\delta w^t \overline{J}_{ww}\delta w + \delta x^t \overline{J}_{xw}\delta w.\end{aligned}$$

*To find the locally optimal feedback law for **u** in period **N**:*

$$\nabla_{u_N}\hat{J} = 0$$

Static Dynamic Control

Result:

$$\delta u_N(\delta x, \delta w) = -\overline{J}_{uu}^{-1}\overline{J}_u - \overline{J}_{uu}^{-1}\overline{J}_{xu}^t\delta x - \overline{J}_{uu}^{-1}\overline{J}_{wu}^t\delta w$$

$$\delta u_N(\delta x, \delta w) = \alpha_N + \beta_N\delta x + \gamma_N\delta w$$

Static Dynamic Control

Result:

$$\delta u_N(\delta x, \delta w) = -\overline{J}_{uu}^{-1}\overline{J}_u - \overline{J}_{uu}^{-1}\overline{J}_{xu}^{t}\delta x - \overline{J}_{uu}^{-1}\overline{J}_{wu}^{t}\delta w$$

$$\delta u_N(\delta x, \delta w) = \alpha_N + \beta_N \delta x + \gamma_N \delta w$$

This feedback law gives the optimal control ***u*** for period ***N*** given a perturbation in the state ***x*** and a perturbation in the static control ***w*** all relative to a nominal (sub-optimal) trajectory.

Static Dynamic Control

By substituting the local optimal feedback law

$$\delta u_N(\delta x, \delta w) = \alpha_N + \beta_N \delta x + \gamma_N \delta w$$

into the original Taylor series for ***J*** we can eliminate δu_N

$$\begin{aligned}
\hat{J}^*(\delta x, \delta w) \;\; = \;\; & (\overline{J} - \frac{1}{2}\overline{J}_u^t \overline{J}_{uu}^{-1} \overline{J}_u) + (\overline{J}_x^t - \overline{J}_u^t \overline{J}_{uu}^{-1} \overline{J}_{xu}^t)\delta x + (\overline{J}_w^t - \overline{J}_u^t \overline{J}_{uu}^{-1} \overline{J}_{wu}^t)\delta w \\
& + \frac{1}{2}\delta x^t (\overline{J}_{xx} - \overline{J}_{xu} \overline{J}_{uu}^{-1} \overline{J}_{xu}^t)\delta x + \frac{1}{2}\delta w^t (\overline{J}_{ww} - \overline{J}_{wu} \overline{J}_{uu}^{-1} \overline{J}_{wu}^t)\delta w \\
& + \delta x^t (\overline{J}_{xw} - \overline{J}_{xu} \overline{J}_{uu}^{-1} \overline{J}_{wu}^t)\delta w.
\end{aligned}$$

Static Dynamic Control

Substituting the local optimal feedback law

$$\delta u_N(\delta x, \delta w) = \alpha_N + \beta_N \delta x + \gamma_N \delta w$$

into the original Taylor series for ***J*** we can eliminate δu_N

$$\begin{aligned}\hat{J}^*(\delta x, \delta w) &= (\overline{J} - \frac{1}{2}\overline{J}_u^t \overline{J}_{uu}^{-1} \overline{J}_u) + (\overline{J}_x^t - \overline{J}_u^t \overline{J}_{uu}^{-1} \overline{J}_{xu}^t)\delta x + (\overline{J}_w^t - \overline{J}_u^t \overline{J}_{uu}^{-1} \overline{J}_{wu}^t)\delta w \\ &+ \frac{1}{2}\delta x^t (\overline{J}_{xx} - \overline{J}_{xu} \overline{J}_{uu}^{-1} \overline{J}_{xu}^t)\delta x + \frac{1}{2}\delta w^t (\overline{J}_{ww} - \overline{J}_{wu} \overline{J}_{uu}^{-1} \overline{J}_{wu}^t)\delta w \\ &+ \delta x^t (\overline{J}_{xw} - \overline{J}_{xu} \overline{J}_{uu}^{-1} \overline{J}_{wu}^t)\delta w.\end{aligned}$$

Defining the coefficients to be ***R, Q, S, P, W***, and ***Y***:

$$\hat{J}^*(\delta x, \delta w) = R + Q^t \delta x + S^t \delta w + \frac{1}{2}\delta x^t P \delta x + \frac{1}{2}\delta w^t W \delta w + \delta x^t Y \delta w$$

Static Dynamic Control

Substituting the local optimal feedback law

$$\delta u_N(\delta x, \delta w) = \alpha_N + \beta_N \delta x + \gamma_N \delta w$$

into the original Taylor series for ***J*** we can eliminate δu_N

$$\begin{aligned}\hat{J}^*(\delta x, \delta w) &= (\overline{J} - \frac{1}{2}\overline{J}_u^t \overline{J}_{uu}^{-1} \overline{J}_u) + (\overline{J}_x^t - \overline{J}_u^t \overline{J}_{uu}^{-1} \overline{J}_{xu}^t)\delta x + (\overline{J}_w^t - \overline{J}_u^t \overline{J}_{uu}^{-1} \overline{J}_{wu}^t)\delta w \\ &+ \frac{1}{2}\delta x^t (\overline{J}_{xx} - \overline{J}_{xu} \overline{J}_{uu}^{-1} \overline{J}_{xu}^t)\delta x + \frac{1}{2}\delta w^t (\overline{J}_{ww} - \overline{J}_{wu} \overline{J}_{uu}^{-1} \overline{J}_{wu}^t)\delta w \\ &+ \delta x^t (\overline{J}_{xw} - \overline{J}_{xu} \overline{J}_{uu}^{-1} \overline{J}_{wu}^t)\delta w.\end{aligned}$$

Defining the coefficients to be ***R, Q, S, P, W***, and ***Y***:

$$\hat{J}^*(\delta x, \delta w) = R + Q^t \delta x + S^t \delta w + \frac{1}{2}\delta x^t P \delta x + \frac{1}{2}\delta w^t W \delta w + \delta x^t Y \delta w$$

Optimal value of J at time t_{N-1} given the state and static control

Static Dynamic Control

Now, proceed to period ***N-1***. Define the ***J*** for period ***N-1***:

$$J(x(t), u_{N-1}, w, t) \doteq \int_{t}^{t_{N-1}} F(x(\tau), u_{N-1}, w, \tau) d\tau$$

*Integral cost over period **N-1***

Static Dynamic Control

Now, proceed to period ***N-1***. Define the ***J*** for period ***N-1***:

$$J(x(t), u_{N-1}, w, t) \doteq \int_t^{t_{N-1}} F(x(\tau), u_{N-1}, w, \tau) d\tau$$

$$+G(x(t_{N-1}), u_{N-1}, w, t_{N-1}, N-1)$$

*End of period **N-1** Point in Time cost*

Static Dynamic Control

Now, proceed to period ***N-1***. Define the ***J*** for period ***N-1***:

$$J(x(t), u_{N-1}, w, t) \doteq \int_t^{t_{N-1}} F(x(\tau), u_{N-1}, w, \tau) d\tau$$

$$+G(x(t_{N-1}), u_{N-1}, w, t_{N-1}, N-1)$$

$$+\hat{J}^*(\delta x, \delta w)$$

*Optimal objective for period **N***

Static Dynamic Control

Now, proceed to period ***N-1***. Define the ***J*** for period ***N-1***:

$$J(x(t), u_{N-1}, w, t) \doteq \int_t^{t_{N-1}} F(x(\tau), u_{N-1}, w, \tau) d\tau$$

$$+G(x(t_{N-1}), u_{N-1}, w, t_{N-1}, N-1)$$

$$+\hat{J}^*(\delta x, \delta w)$$

By analogy to the method used for period ***N*** a locally optimal law can be generated for period ***N-1***:

$$\delta u_{N\text{-}1}(\delta x, \delta w) = \alpha_{N\text{-}1} + \beta_{N\text{-}1} \delta x + \gamma_{N\text{-}1} \delta w$$

Static Dynamic Control

The process is repeated backward to generate locally optimal feedback laws for ***u*** for periods ***N, N-1,…,1***.

Period 1 must be handled differently in order to incorporate the initial condition function and compute the optimal update for the static control vector ***w***.

Static Dynamic Control

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For period 1, the truncated Taylor series is:

$$\begin{aligned}\hat{J}(\delta x, \delta u_1, \delta w) \;&=\; \overline{J} + \overline{J}_x^t \delta x + \overline{J}_u^t \delta u_1 + \overline{J}_w^t \delta w + \frac{1}{2}\delta x^t \overline{J}_{xx} \delta x + \delta x^t \overline{J}_{xu} \delta u_1 + \delta w^t \overline{J}_{wu} \delta u_1 \\ &+ \frac{1}{2}\delta u_1^t \overline{J}_{uu} \delta u_1 + \frac{1}{2}\delta w^t \overline{J}_{ww} \delta w + \delta x^t \overline{J}_{xw} \delta w,\end{aligned}$$

Static Dynamic Control

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We can eliminate δx using a Taylor series expansion of the initial condition function Γ:

$$\hat{\delta x}(w) = \overline{\Gamma}_w \delta w + \frac{1}{2}\delta w^t \overline{\Gamma}_{ww} \delta w$$

Static Dynamic Control

After δx eliminated:

$$\hat{J}(\delta u_1, \delta w) = \overline{J} + \tilde{\overline{J}}_w^t \delta w + \frac{1}{2}\delta w^t \tilde{\overline{J}}_{ww} \delta w + \overline{J}_u^t \delta u_1 + \frac{1}{2}\delta u_1^t \overline{J}_{uu} \delta u_1 + \delta w^t \tilde{\overline{J}}_{wu} \delta u_1$$

Static Dynamic Control

Eliminating δx :

$$\hat{J}(\delta u_1, \delta w) = \overline{J} + \tilde{\overline{J}}_w^t \delta w + \frac{1}{2} \delta w^t \tilde{\overline{J}}_{ww} \delta w + \overline{J}_u^t \delta u_1 + \frac{1}{2} \delta u_1^t \overline{J}_{uu} \delta u_1 + \delta w^t \tilde{\overline{J}}_{wu} \delta u_1$$

To find the optimal update for both δu and δu we must simultaneously solve

$$\frac{\partial \hat{J}}{\partial u_1} = 0 = \overline{J}_u + \tilde{\overline{J}}_{uw} \delta w^* + \overline{J}_{uu} \delta u_1^*$$

$$\frac{\partial \hat{J}}{\partial w} = 0 = \tilde{\overline{J}}_w + \tilde{\overline{J}}_{uw}^t \delta u_1^* + \tilde{\overline{J}}_{ww} \delta w^*$$

Static Dynamic Control

The locally optimal updates and feedback laws are applied from period 1 forward. The feedback laws are damped if necessary by a parameter $\varepsilon \in (0,1]$ until the trajectory is improved.

$$\delta u_1 = \epsilon \alpha_1$$

$$\delta w = \epsilon \xi$$

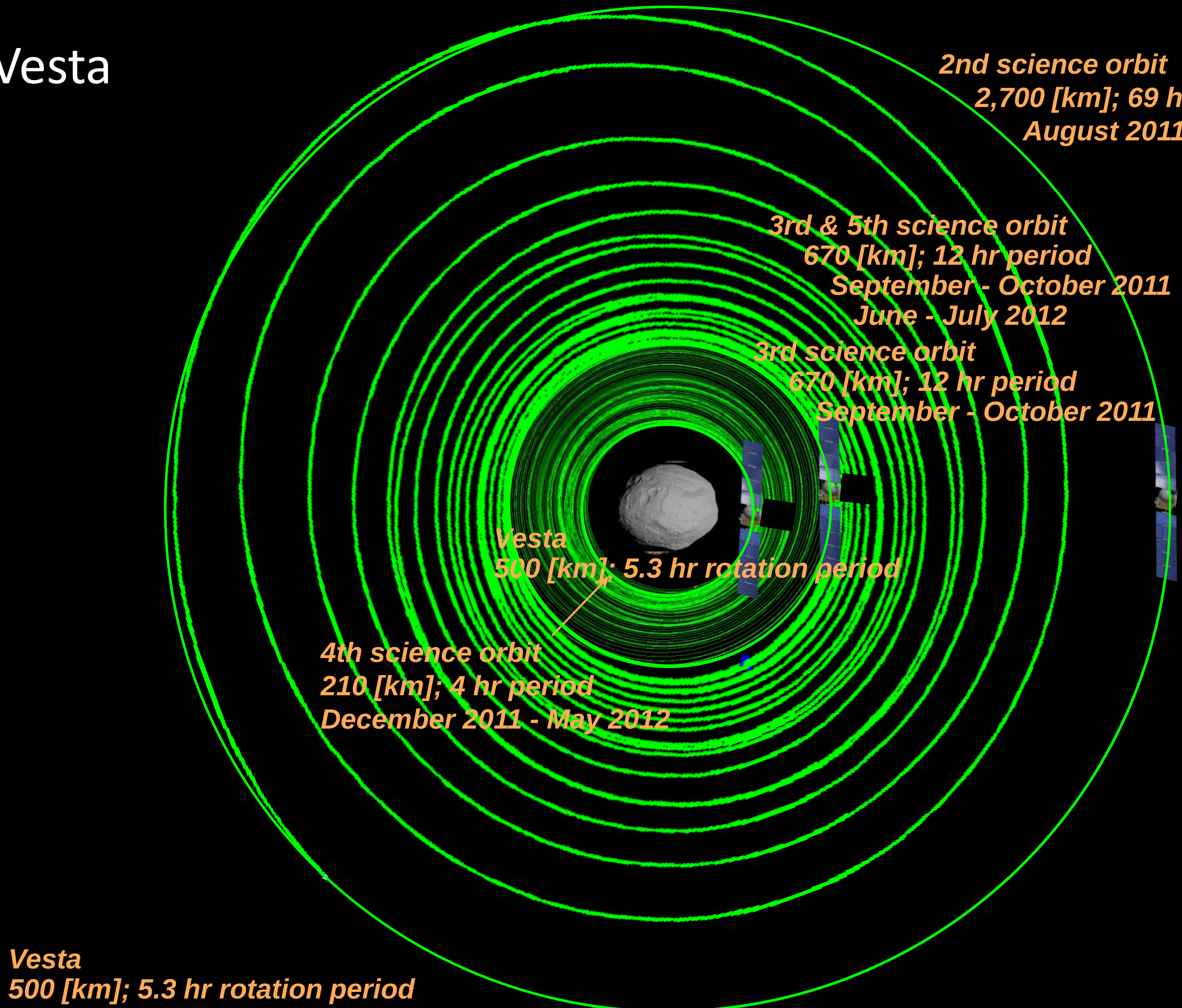
Period 1 updates

$$\delta u_i = \epsilon \alpha_i + \beta_i \delta x + \epsilon \gamma_i \xi$$

Period 2,3,…,N feedback laws

Next new feedback laws are computed and applied to the improved trajectory iteratively until convergence is obtained.

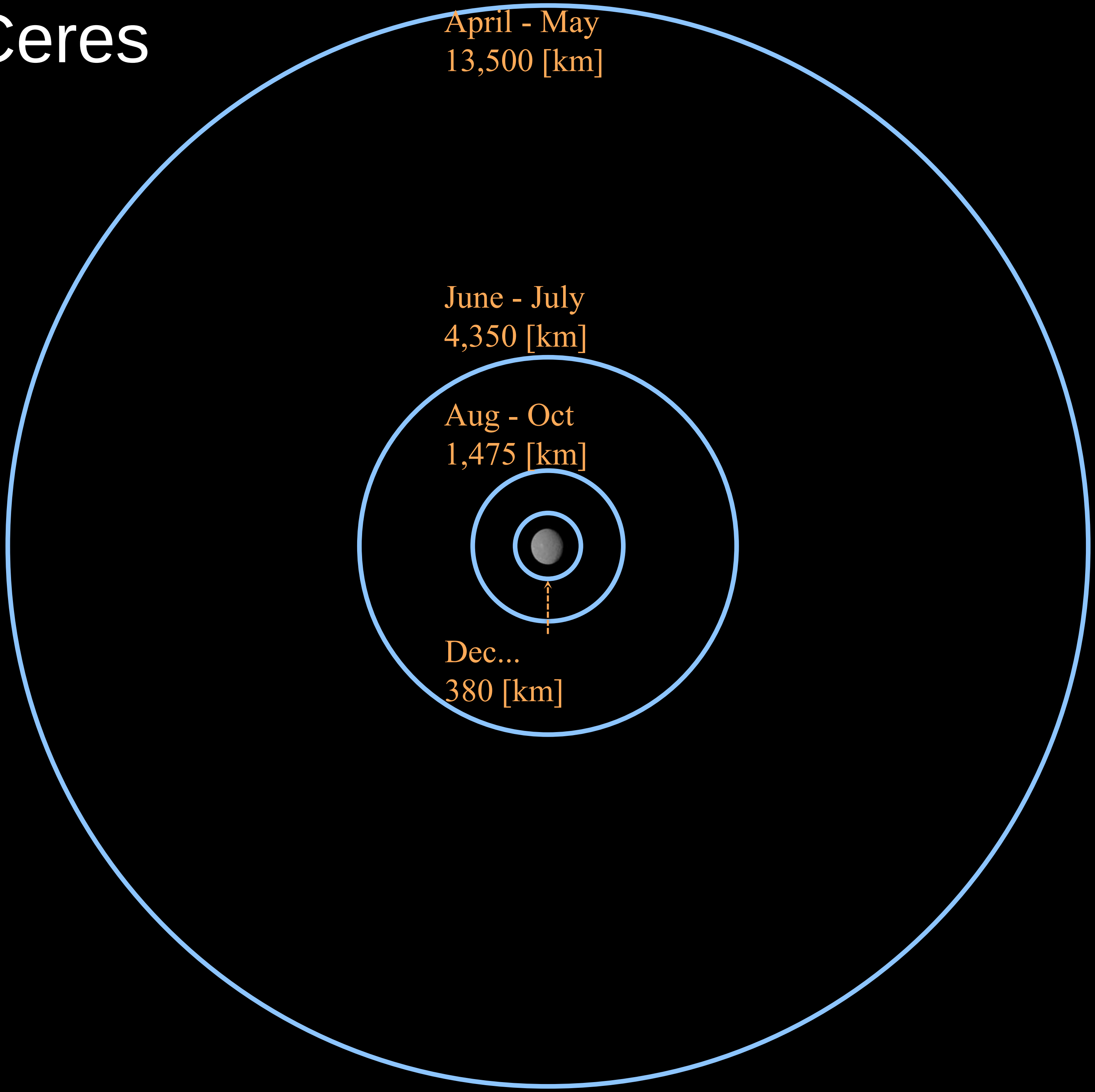
Vesta



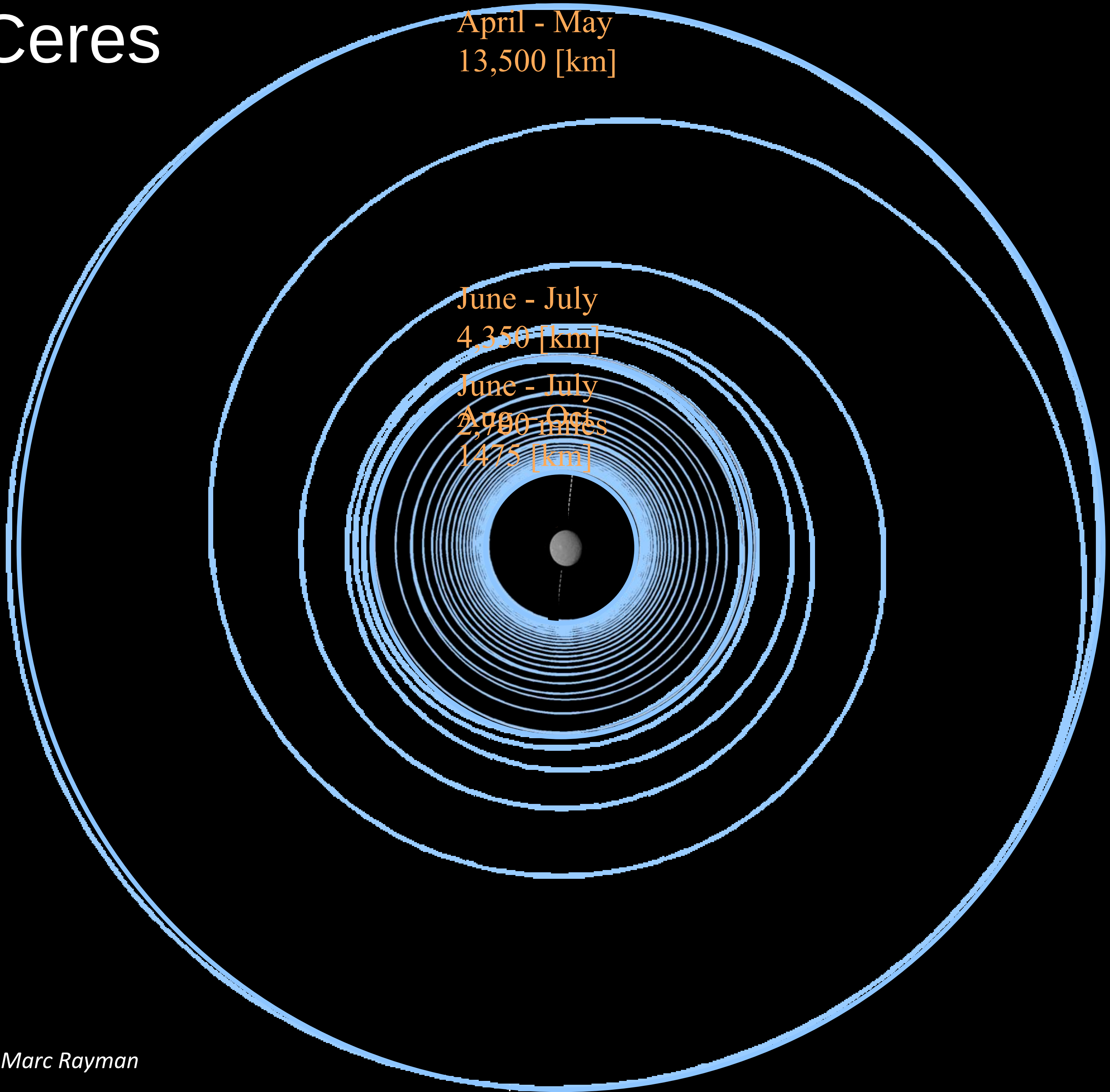
2nd science orbit
2,700 [km]; 69 hr period
August 2011
3rd & 5th science orbit
670 [km]; 12 hr period
September - October 2011
June - July 2012
3rd science orbit
670 [km]; 12 hr period
September - October 2011
Vesta
500 [km]; 5.3 hr rotation period
4th science orbit
210 [km]; 4 hr period
December 2011 - May 2012
Vesta
500 [km]; 5.3 hr rotation period

Vesta Science Orbit Movies

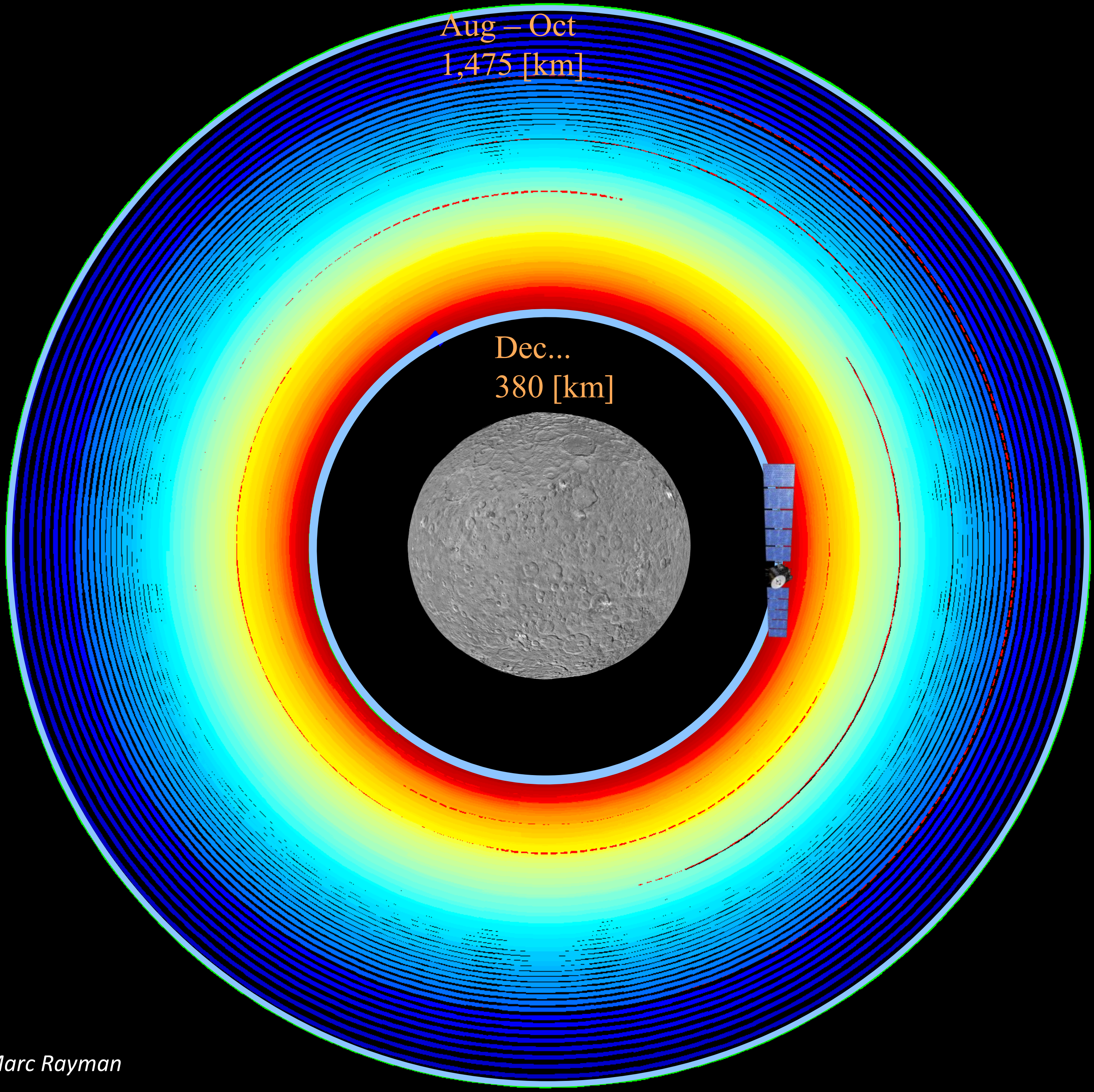
Ceres



Ceres

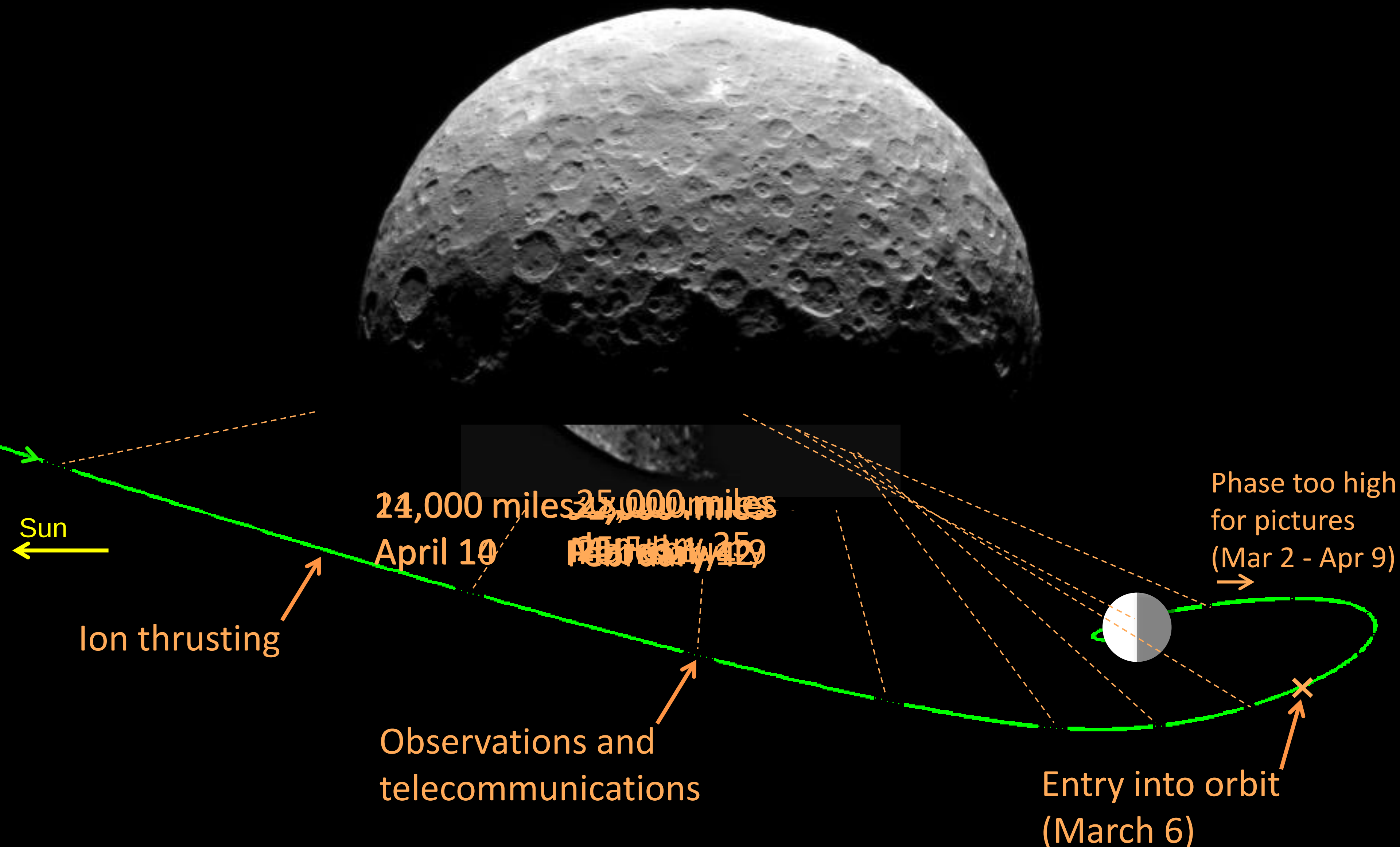


Slide credit: Marc Rayman



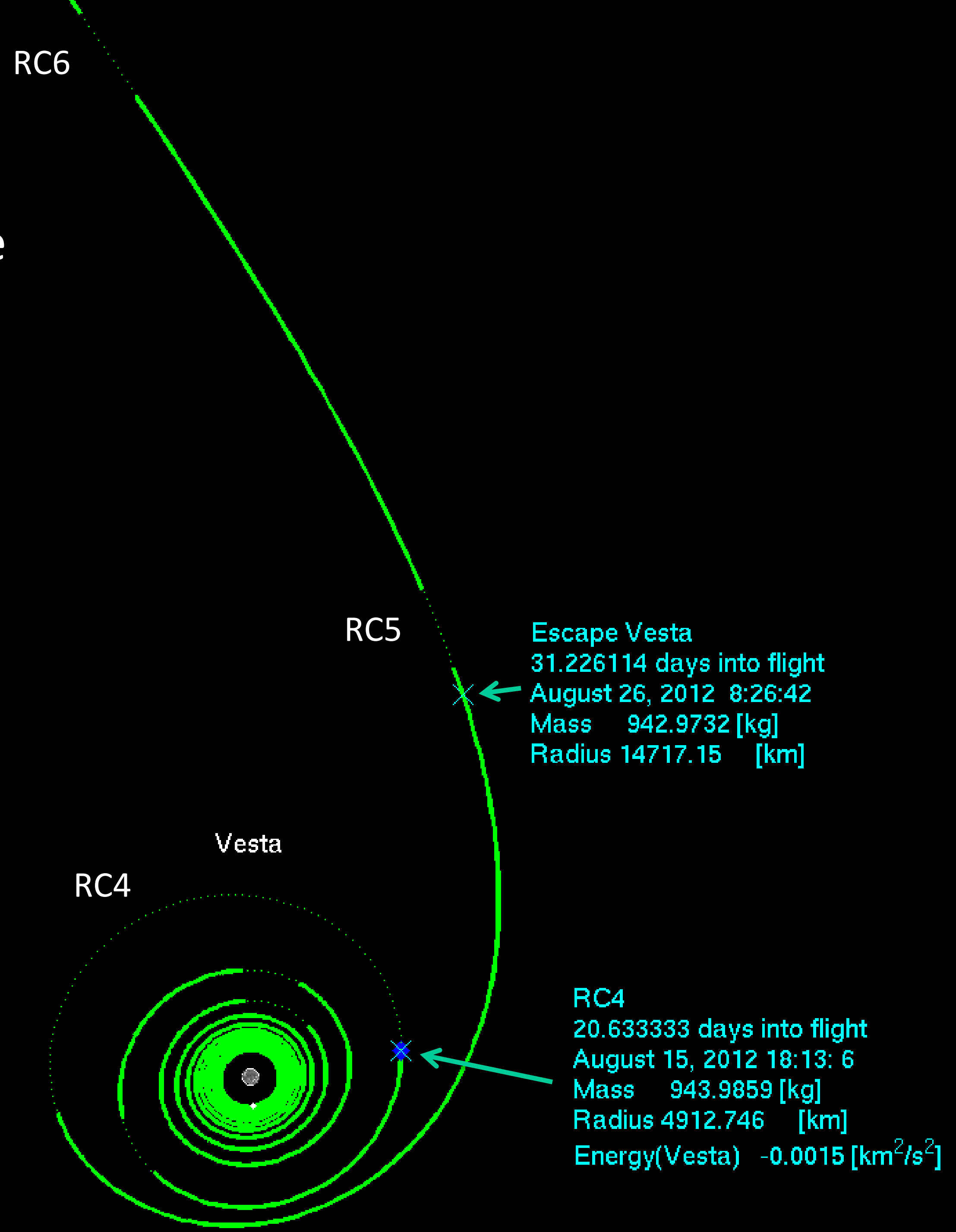
Slide credit: Marc Rayman

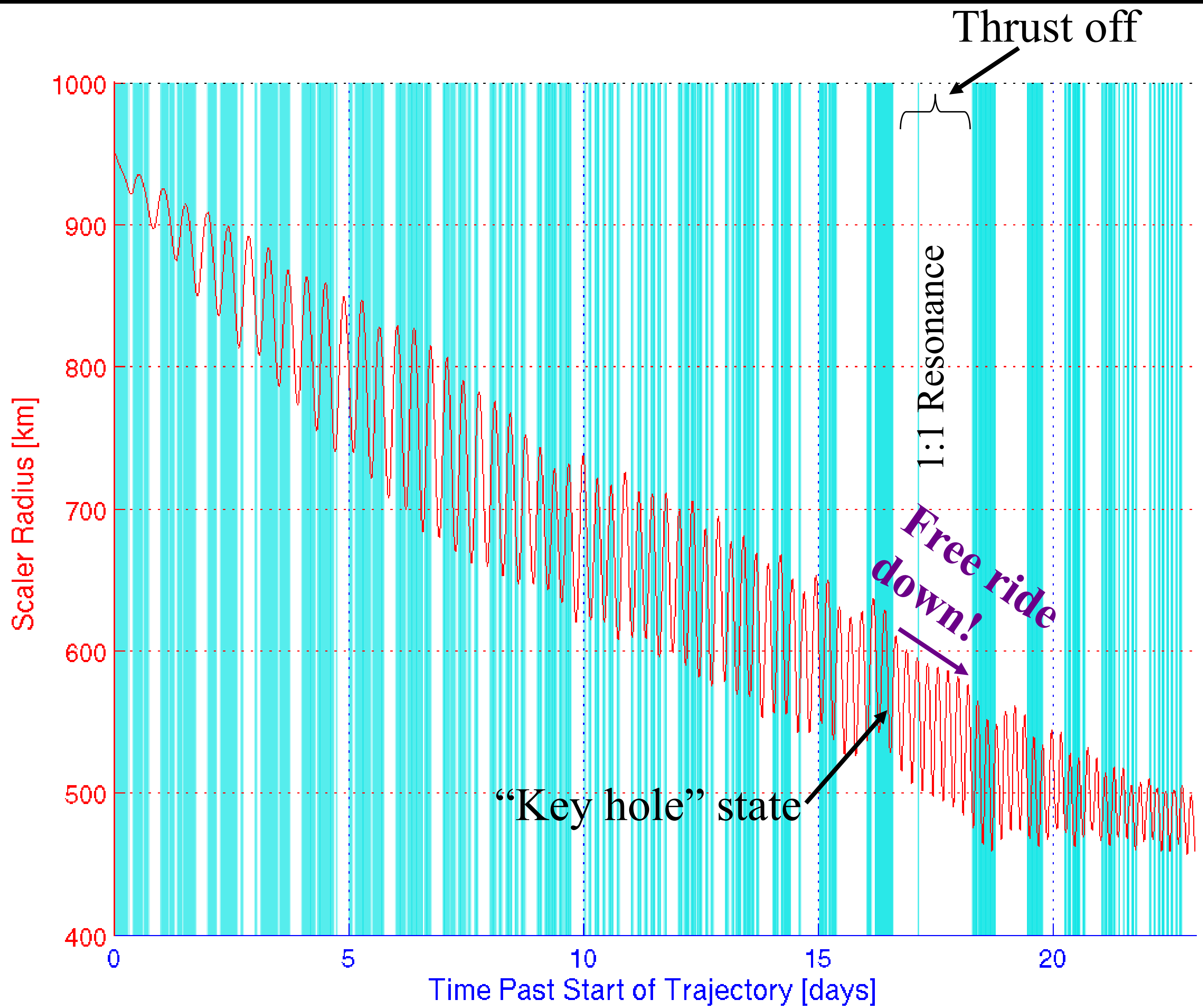
Dawn Approaches



Slide credit: Marc Rayman

Vesta departure science orbit planning

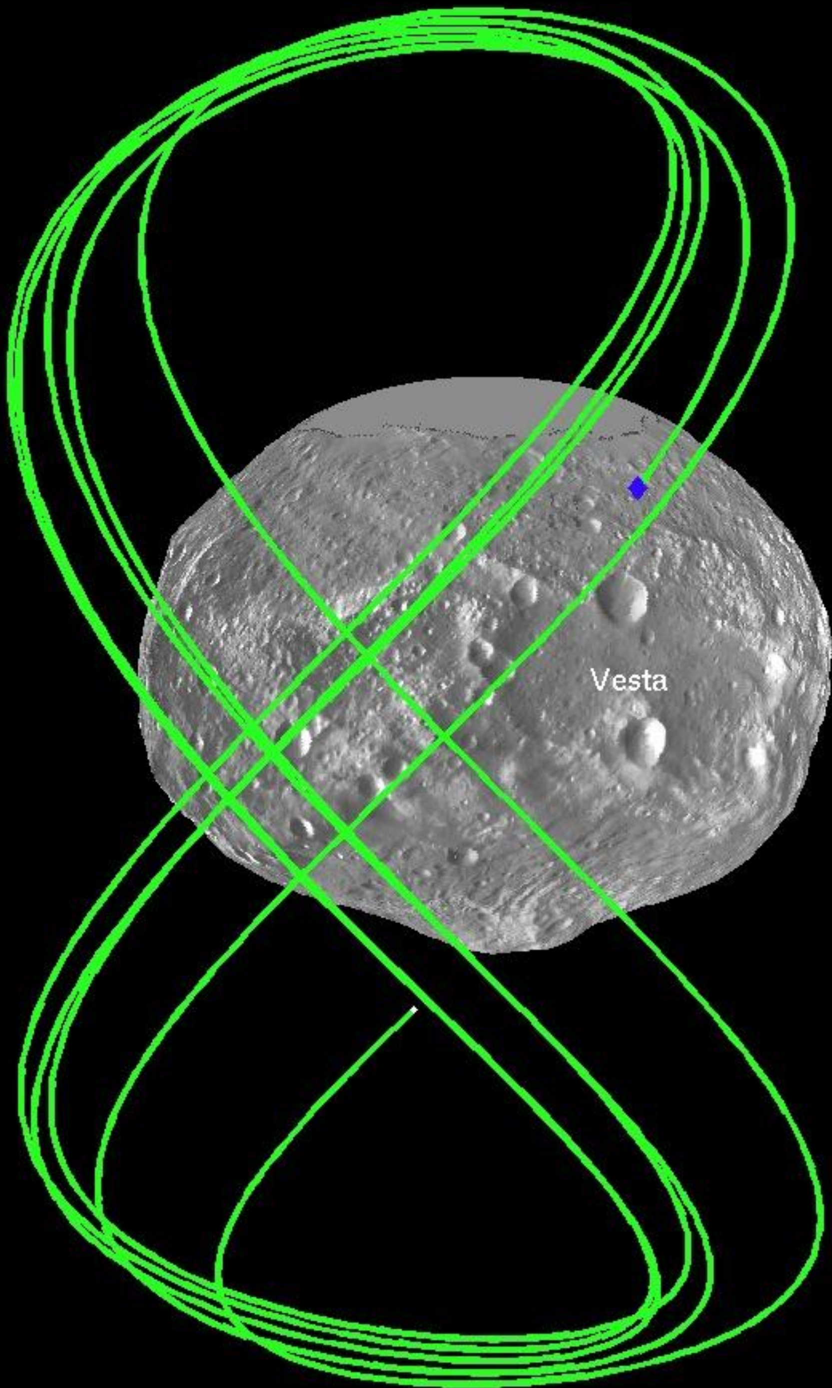




Thrust off
1:1 Resonance
Free ride down!
"Key hole" state
1000
900
800
700
600
500
400
Scaler Radius [km]
0
5
10
15
20
Time Past Start of Trajectory [days]

Vesta Resonance Movie

Mystic



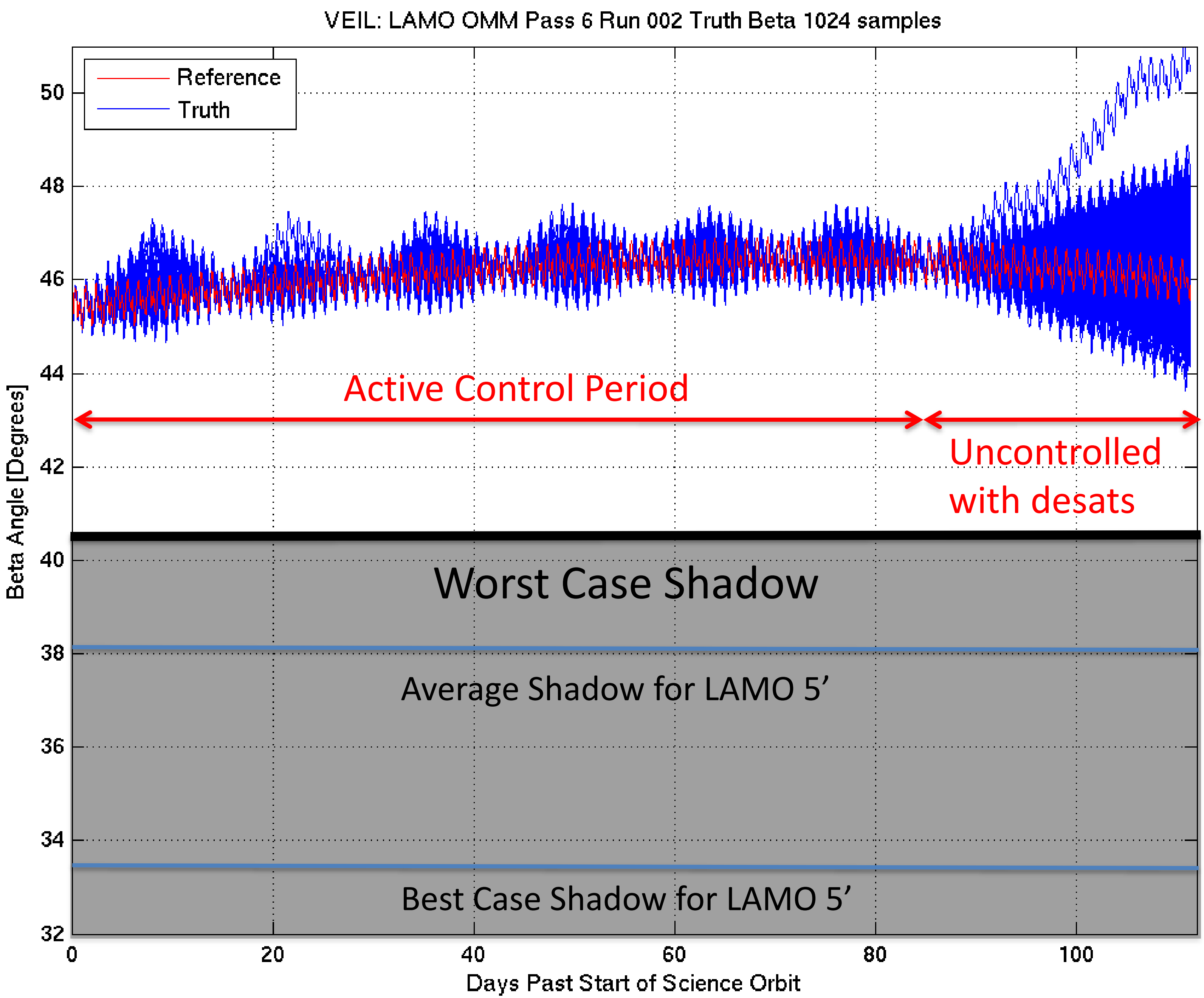
What if we missed the keyhole state?

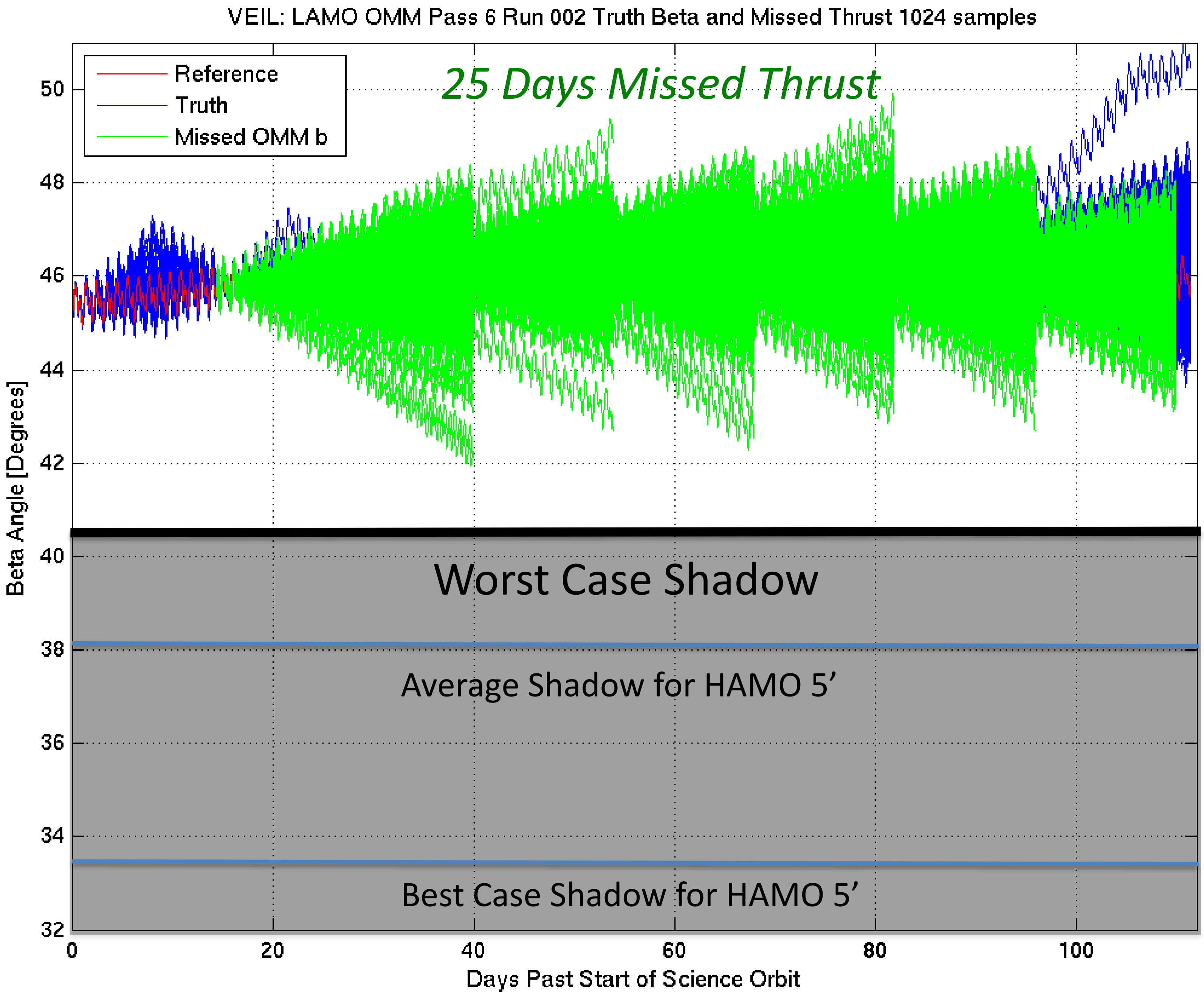
- Spacecraft can "bounce": thrust fails to reduce the orbital radius:
  Ion thruster energy → Vesta rotational energy

- Spacecraft can be driven up when you are trying to go down:
  Vesta rotational energy → Spacecraft orbital energy

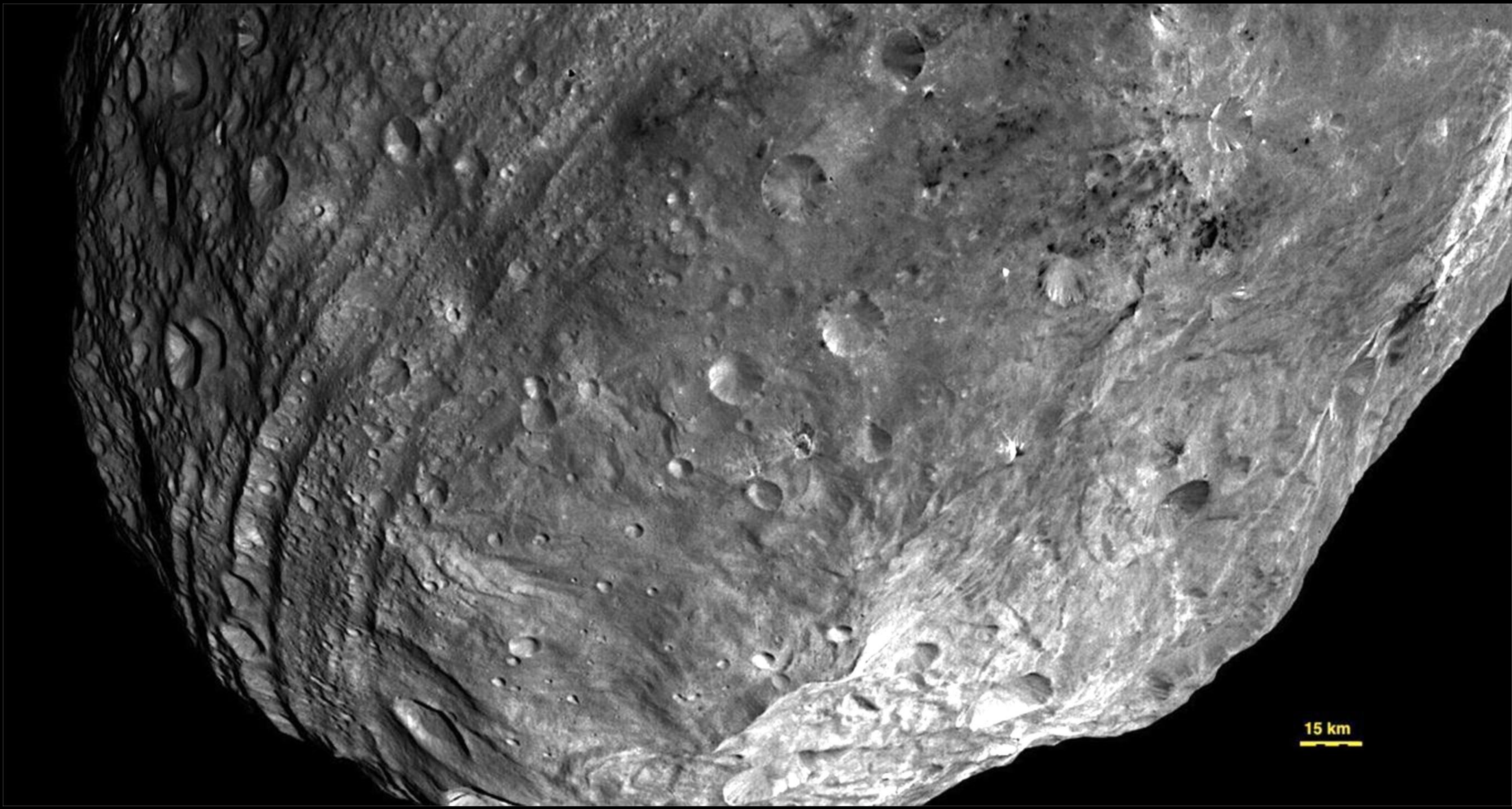
- The spacecraft orbit can have its plane torqued around
  Vesta angular momentum → Spacecraft angular momentum

This is particularly dangerous for Dawn


VEIL: LAMO OMM Pass 6 Run 002 Truth Beta 1024 samples
Reference
Truth
Beta Angle [Degrees]
50
48
46
44
42
40
38
36
34
32
0
20
40
60
80
100
Days Past Start of Science Orbit
Active Control Period
Uncontrolled with desats
Worst Case Shadow
Average Shadow for LAMO 5’
Best Case Shadow for LAMO 5’


VEIL: LAMO OMM Pass 6 Run 002 Truth Beta and Missed Thrust 1024 samples
Reference
Truth
Missed OMM b
25 Days Missed Thrust
Worst Case Shadow
Average Shadow for HAMO 5'
Best Case Shadow for HAMO 5'
Beta Angle [Degrees]
Days Past Start of Science Orbit
50
48
46
44
42
40
38
36
34
32
0
20
40
60
80
100

Vesta



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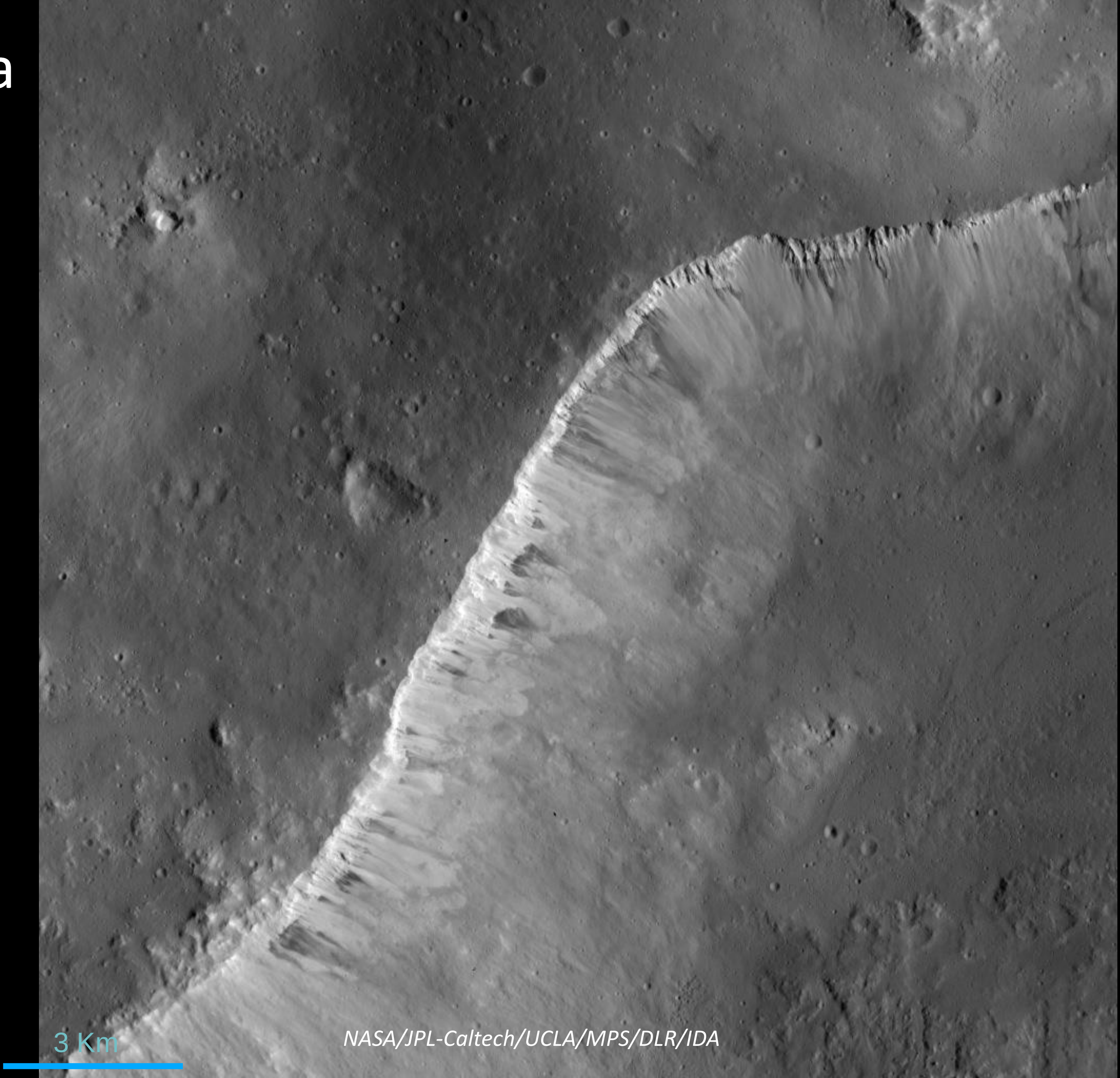
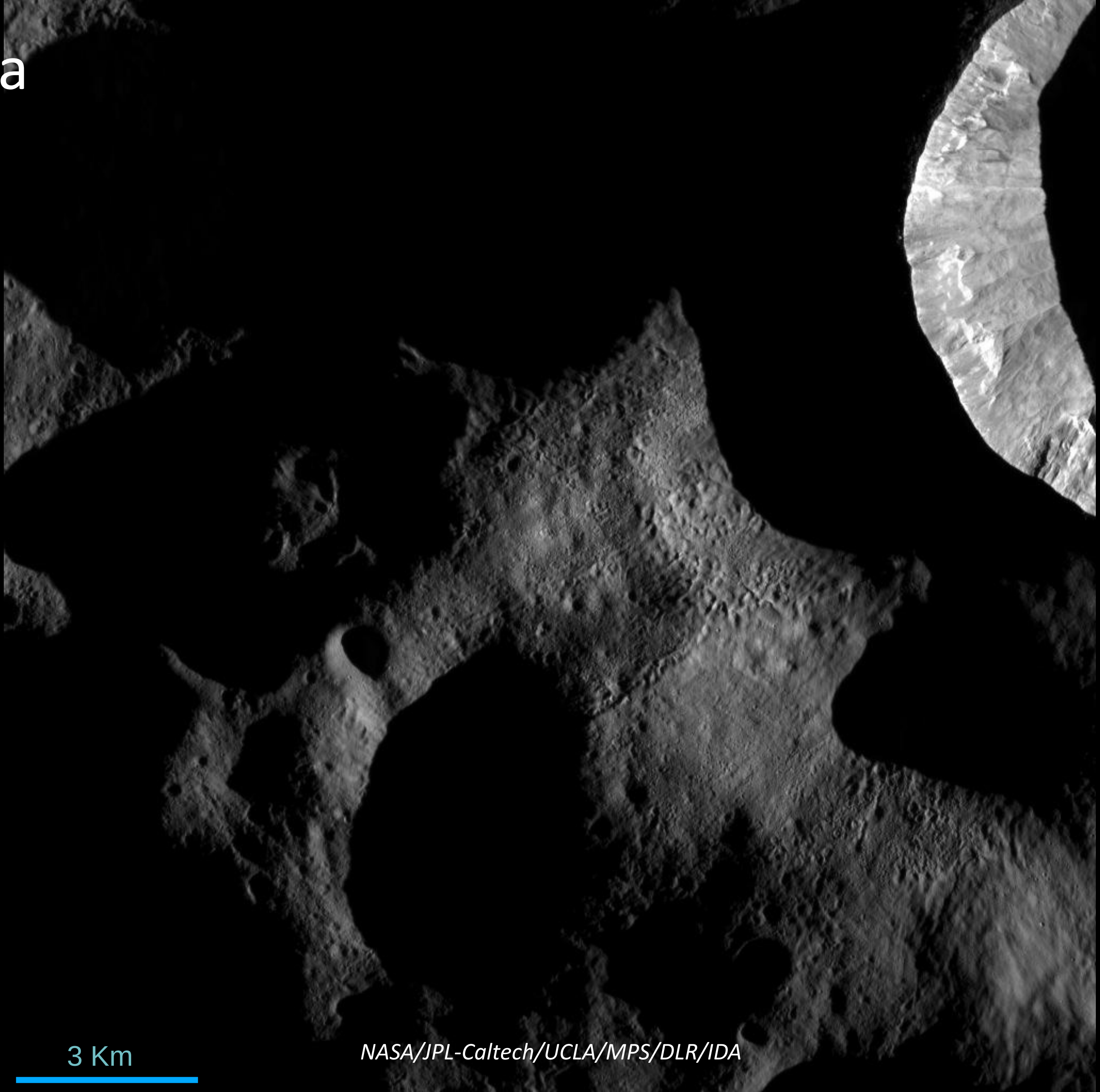
Vesta



Vesta



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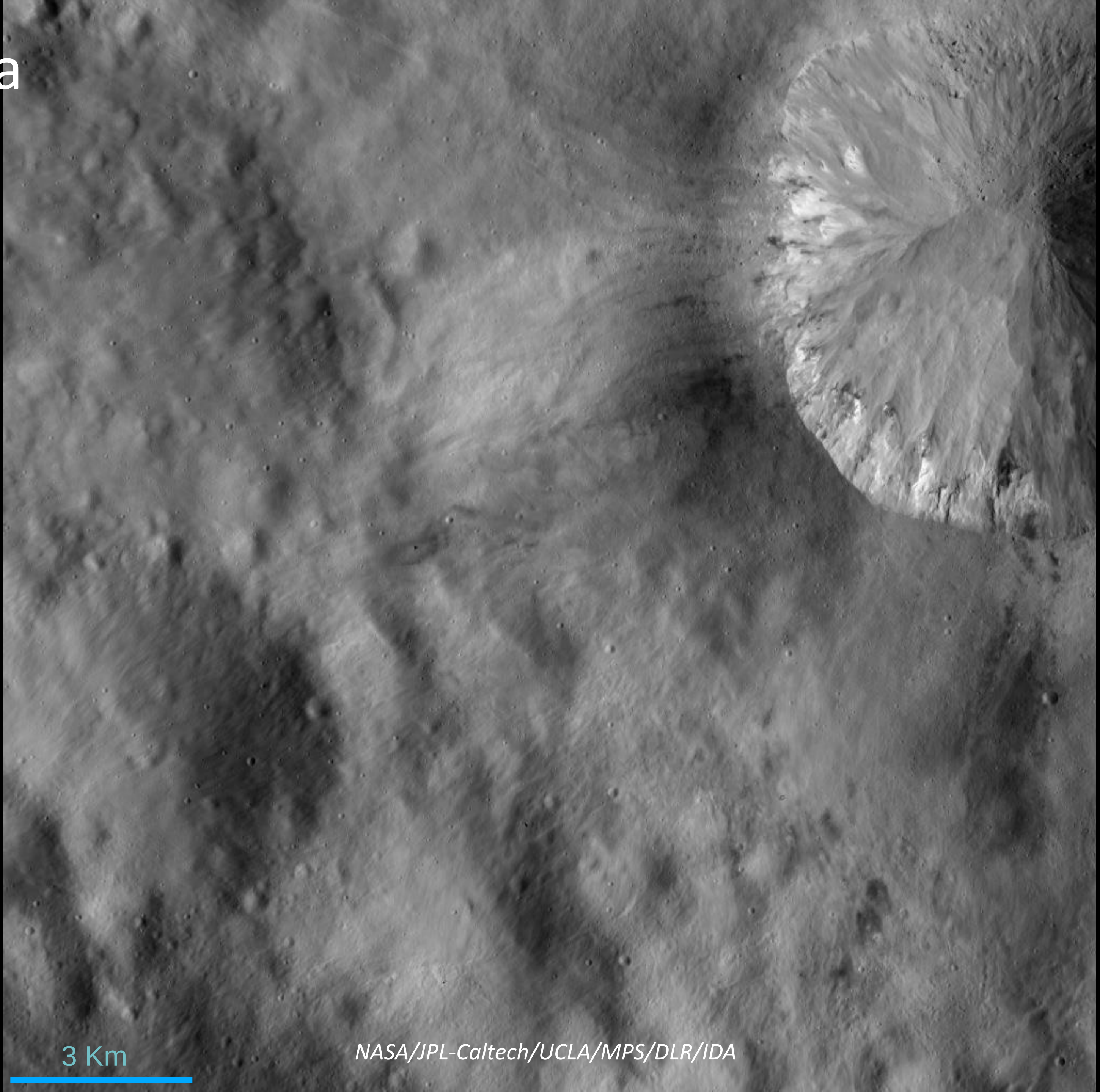
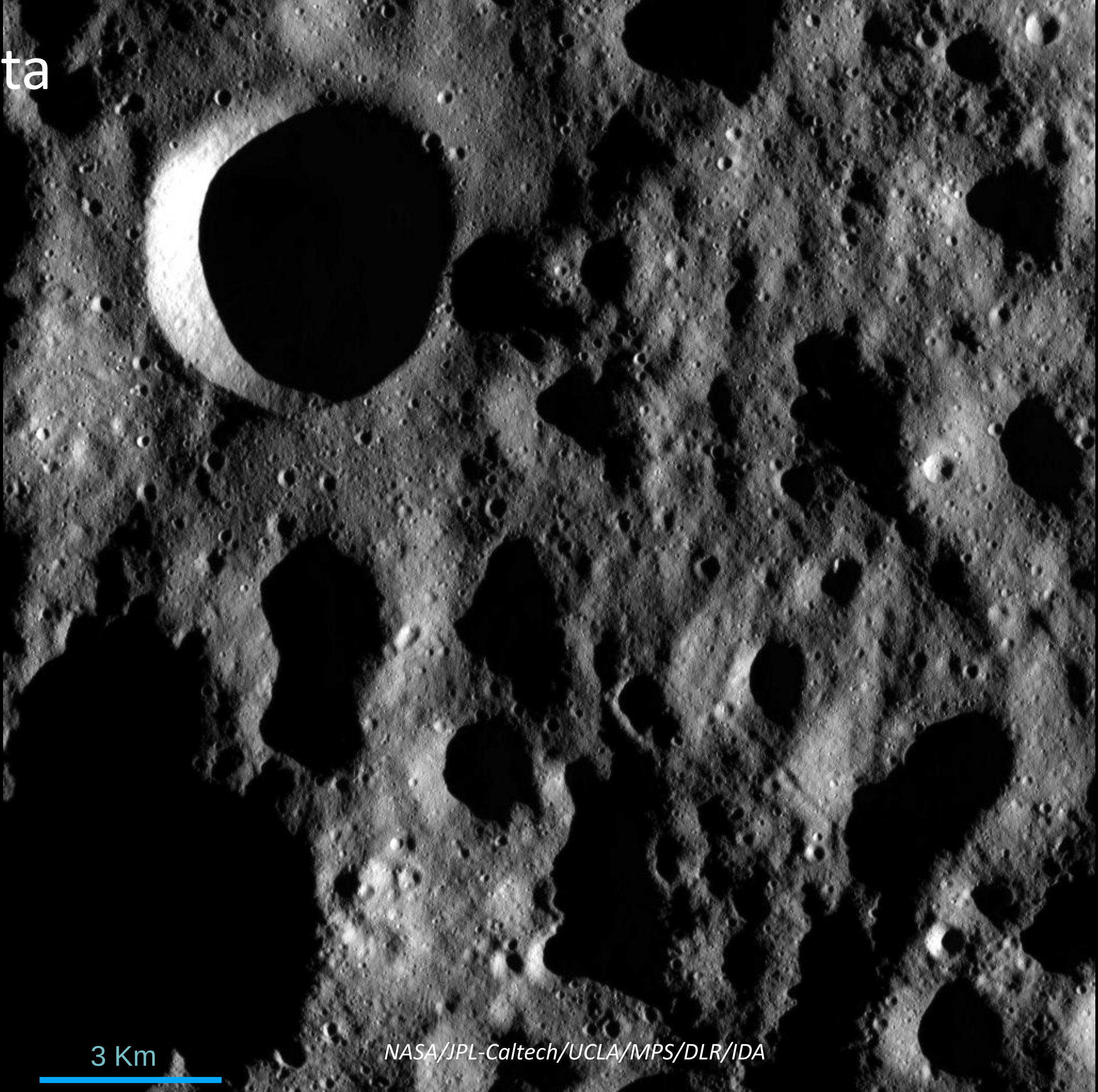
Vesta

Vesta

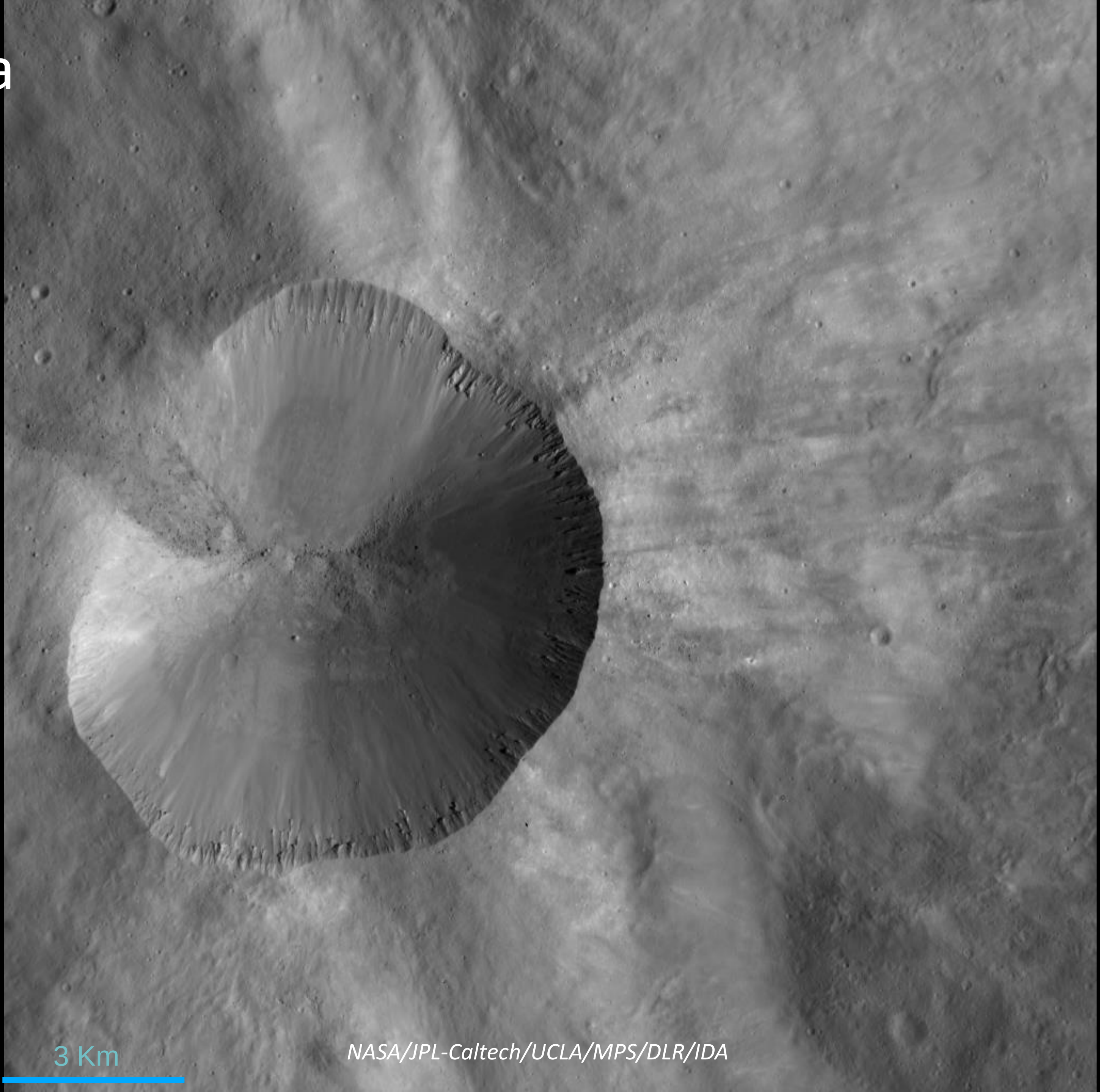


NASA/JPL-Caltech/UCLA/MPS/DLR/IDA

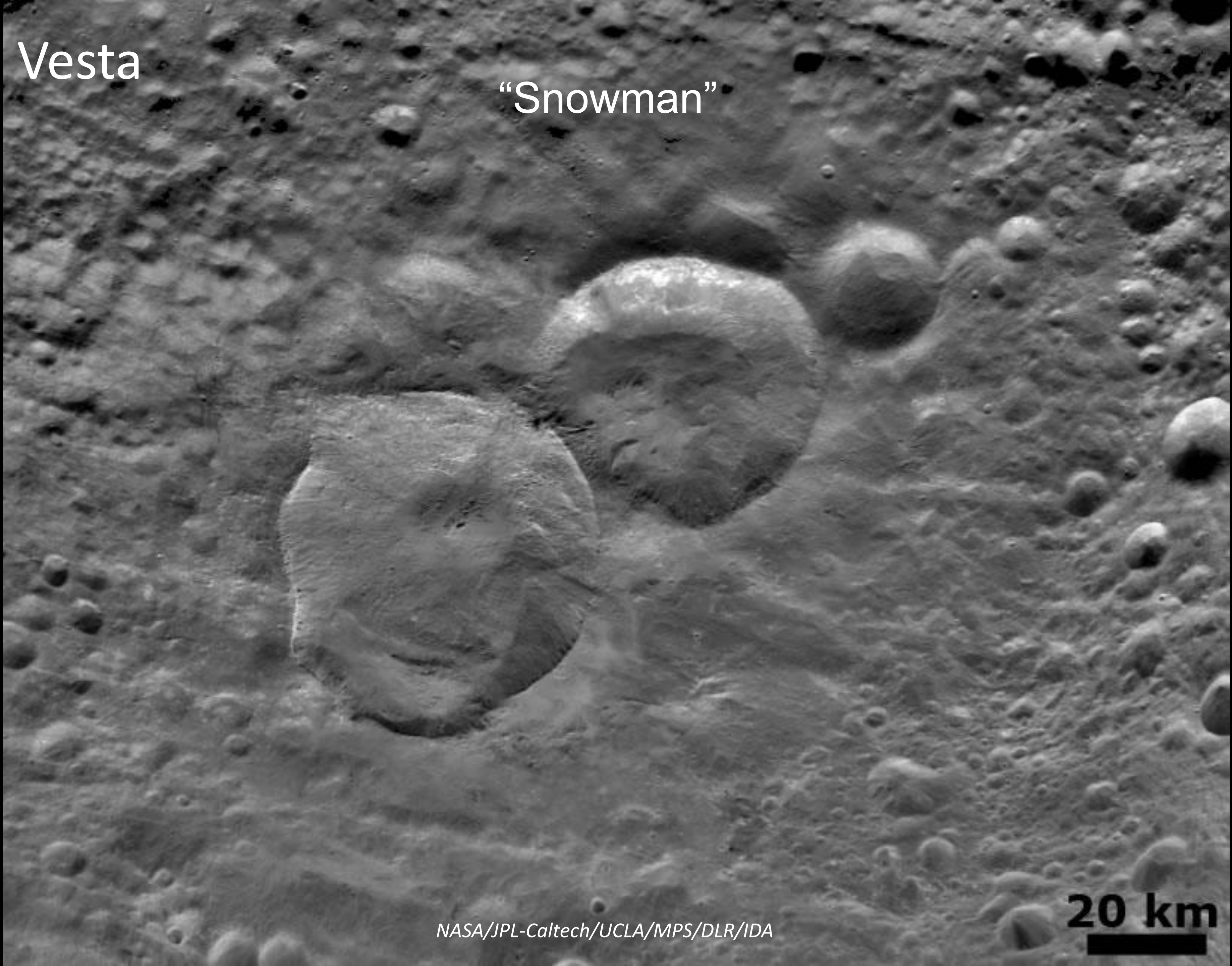
Vesta

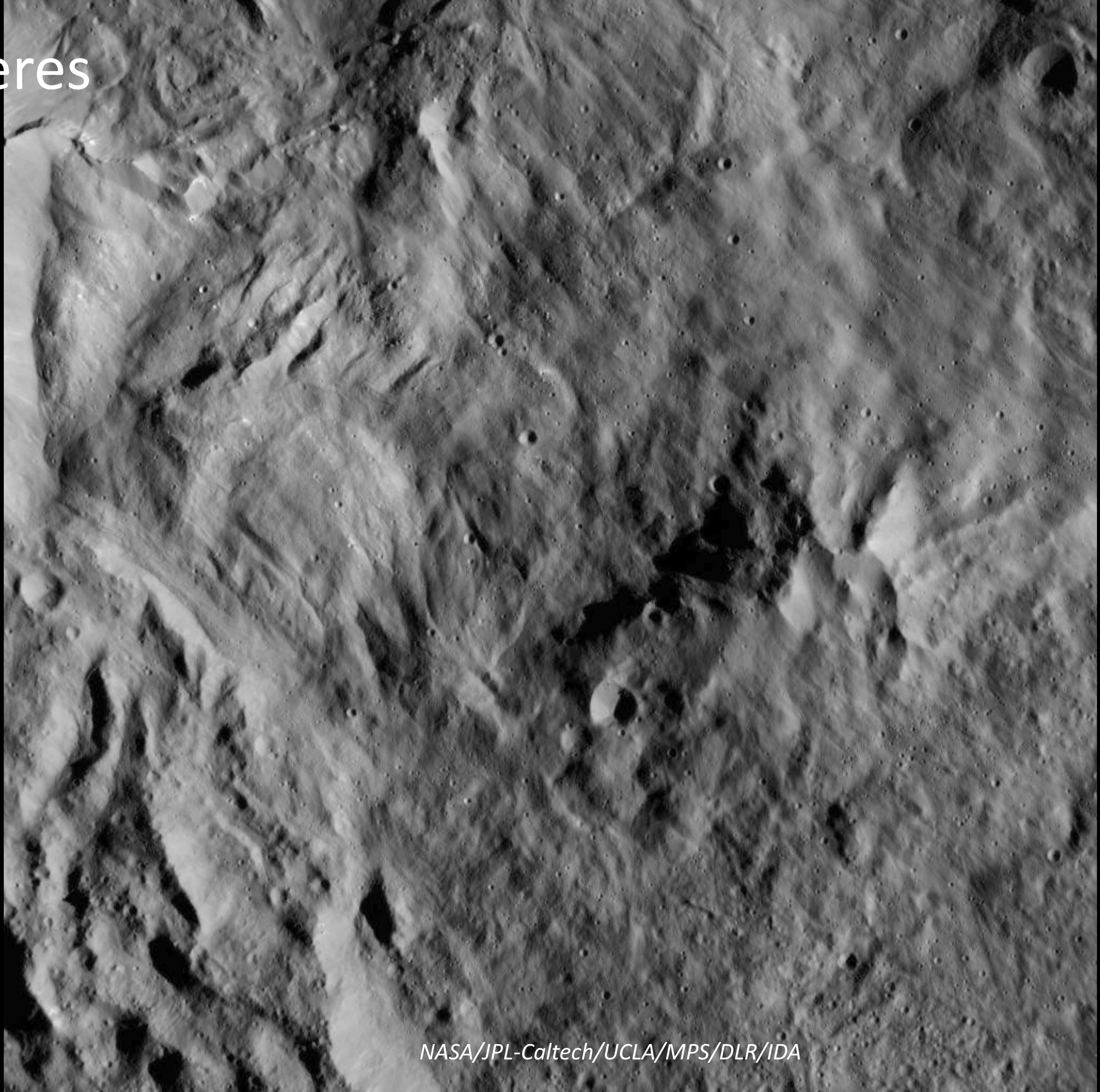


Vesta



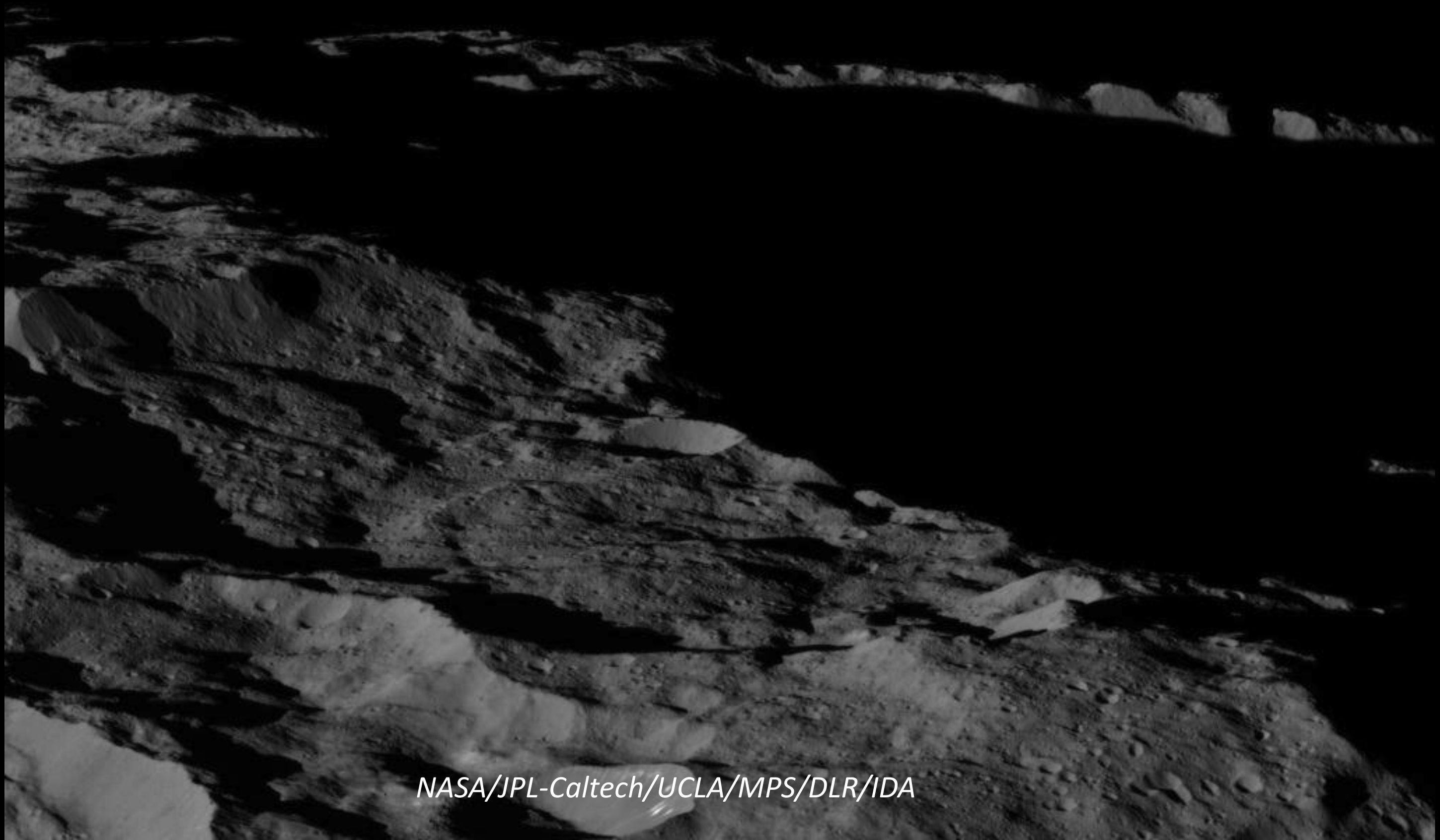
NASA/JPL-Caltech/UCLA/MPS/DLR/IDA


Vesta
“Snowman”
20 km
NASA/JPL-Caltech/UCLA/MPS/DLR/IDA


Ceres
NASA/JPL-Caltech/UCLA/MPS/DLR/IDA

Ceres

Ceres



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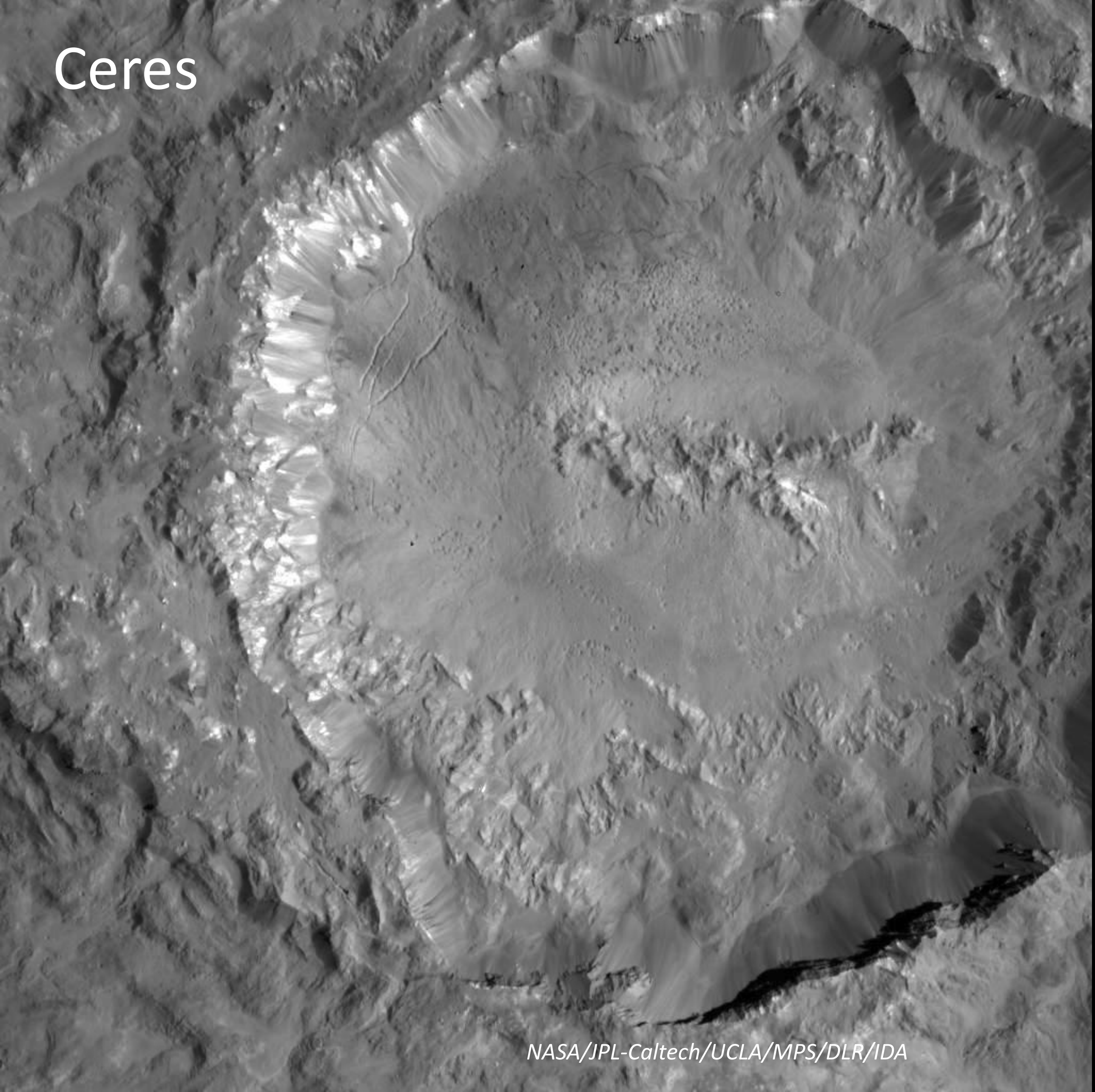
Ceres



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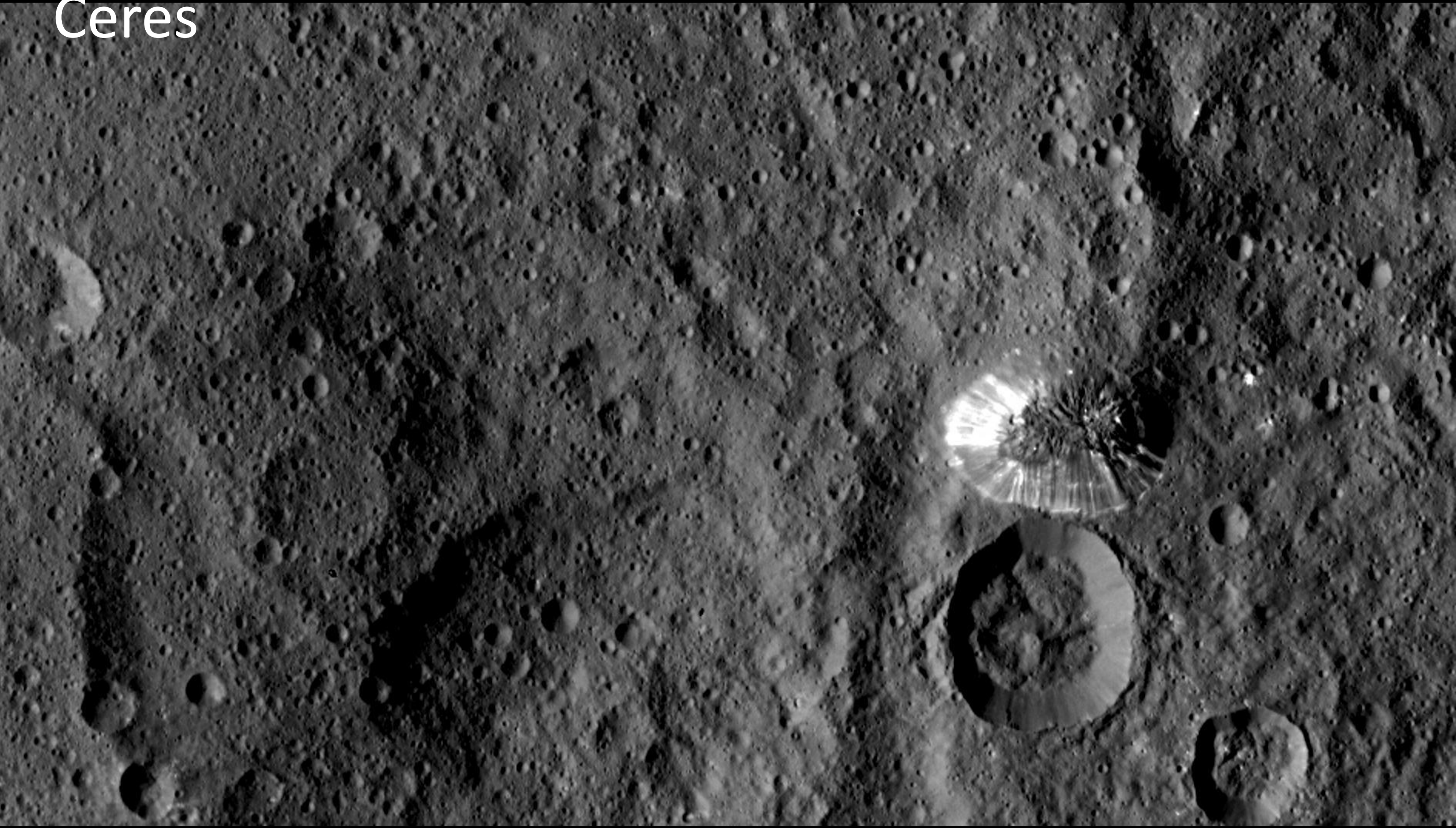
Ceres



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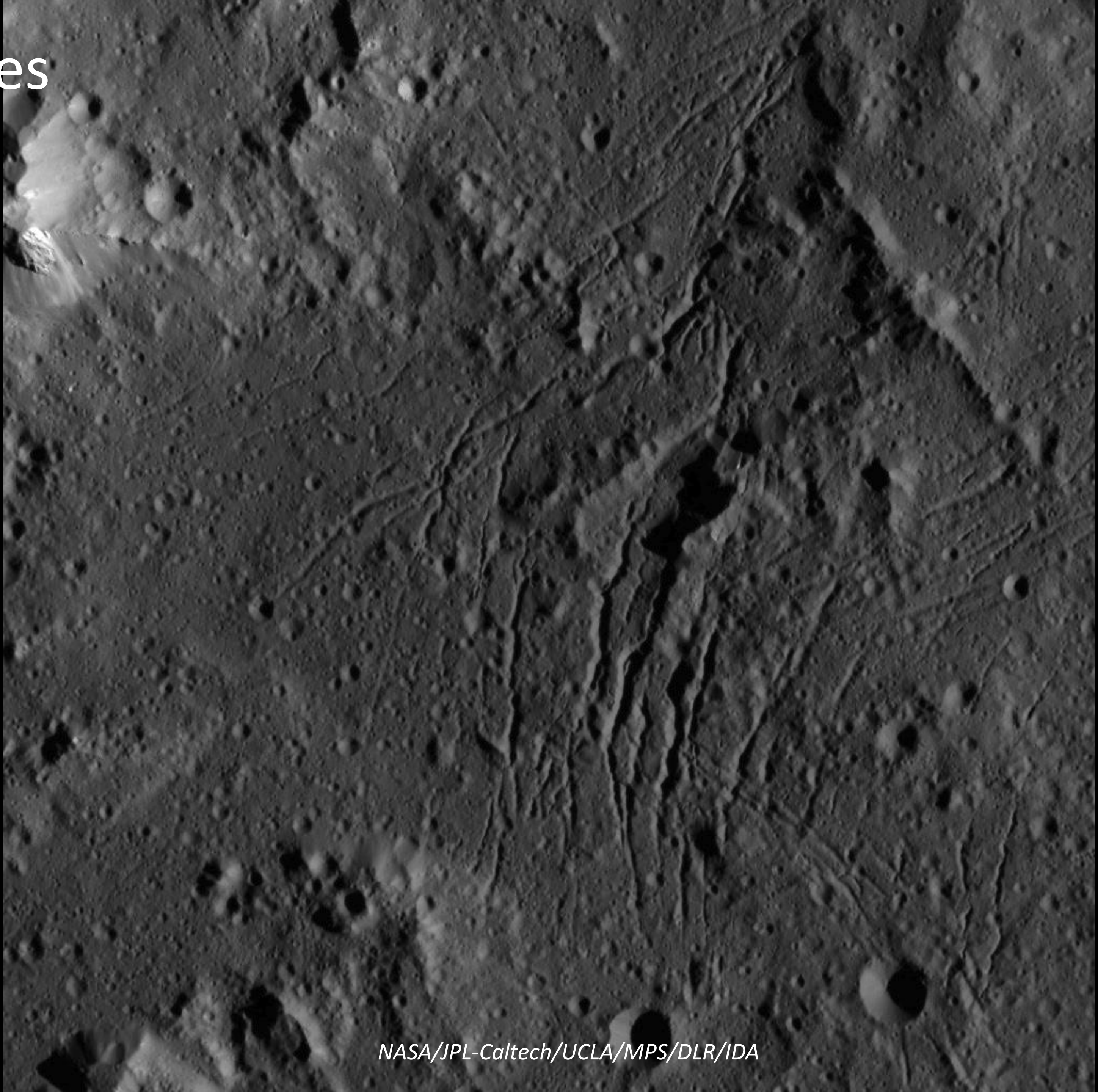
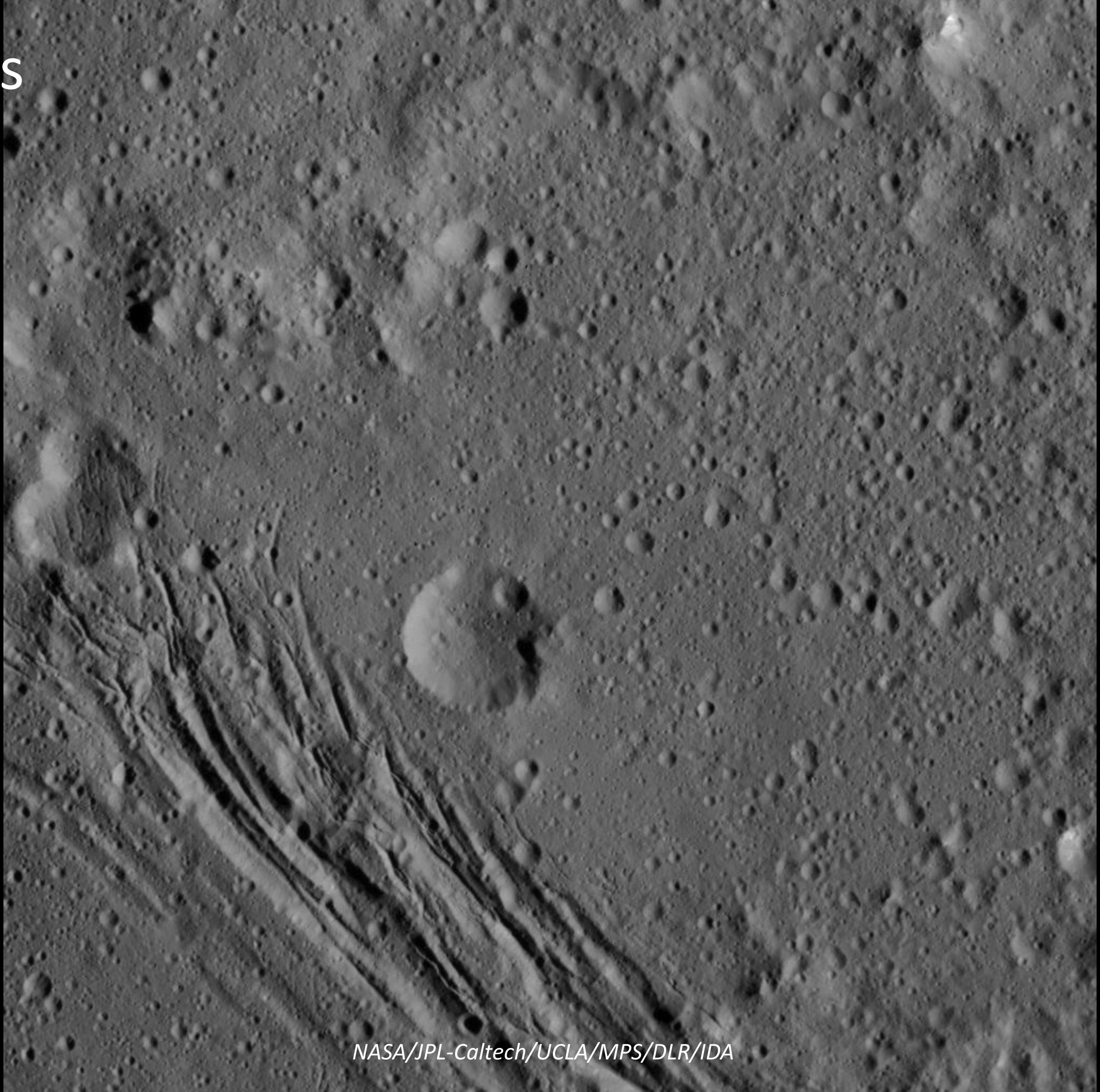
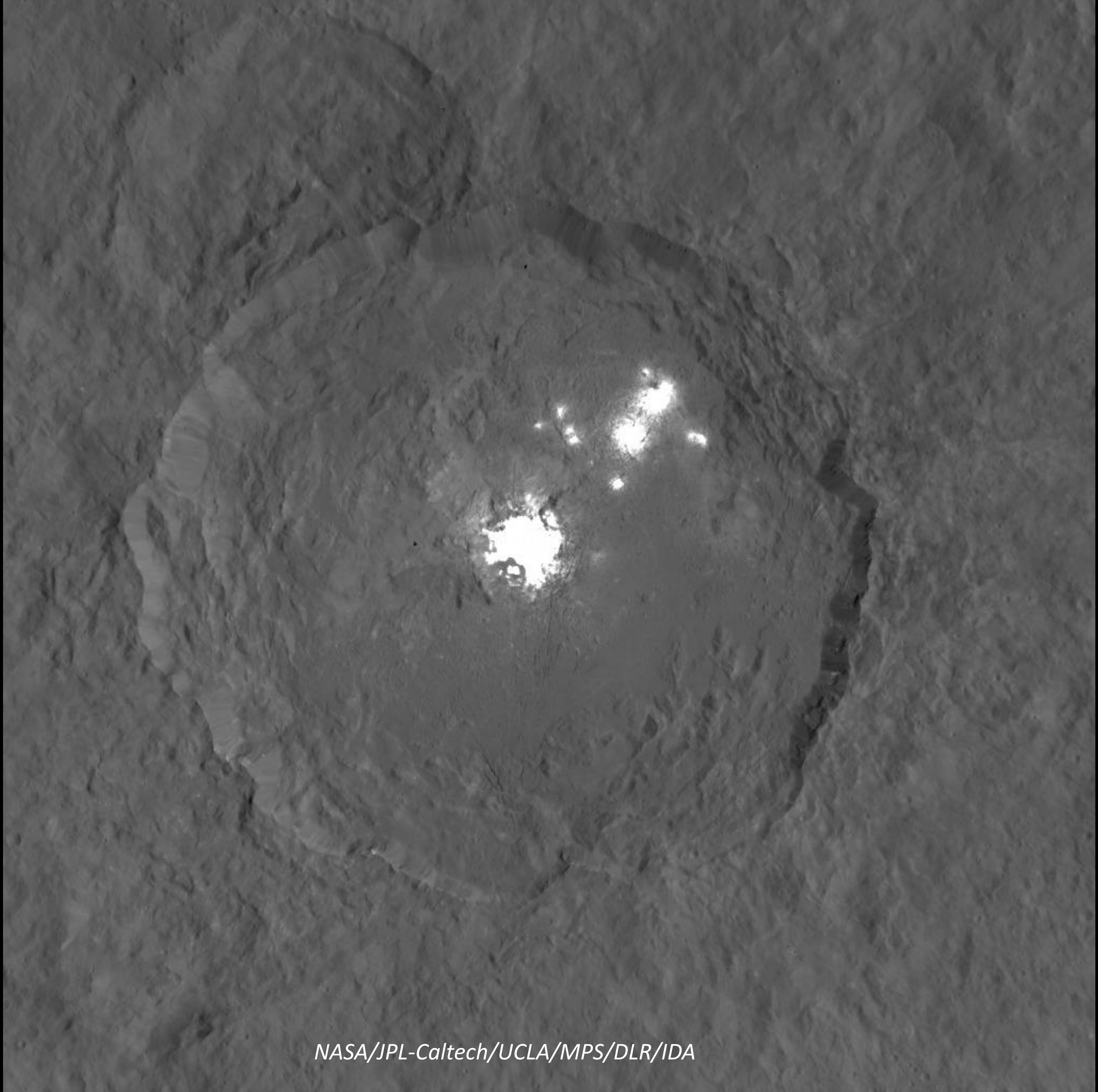
Ceres



Ceres



Ceres



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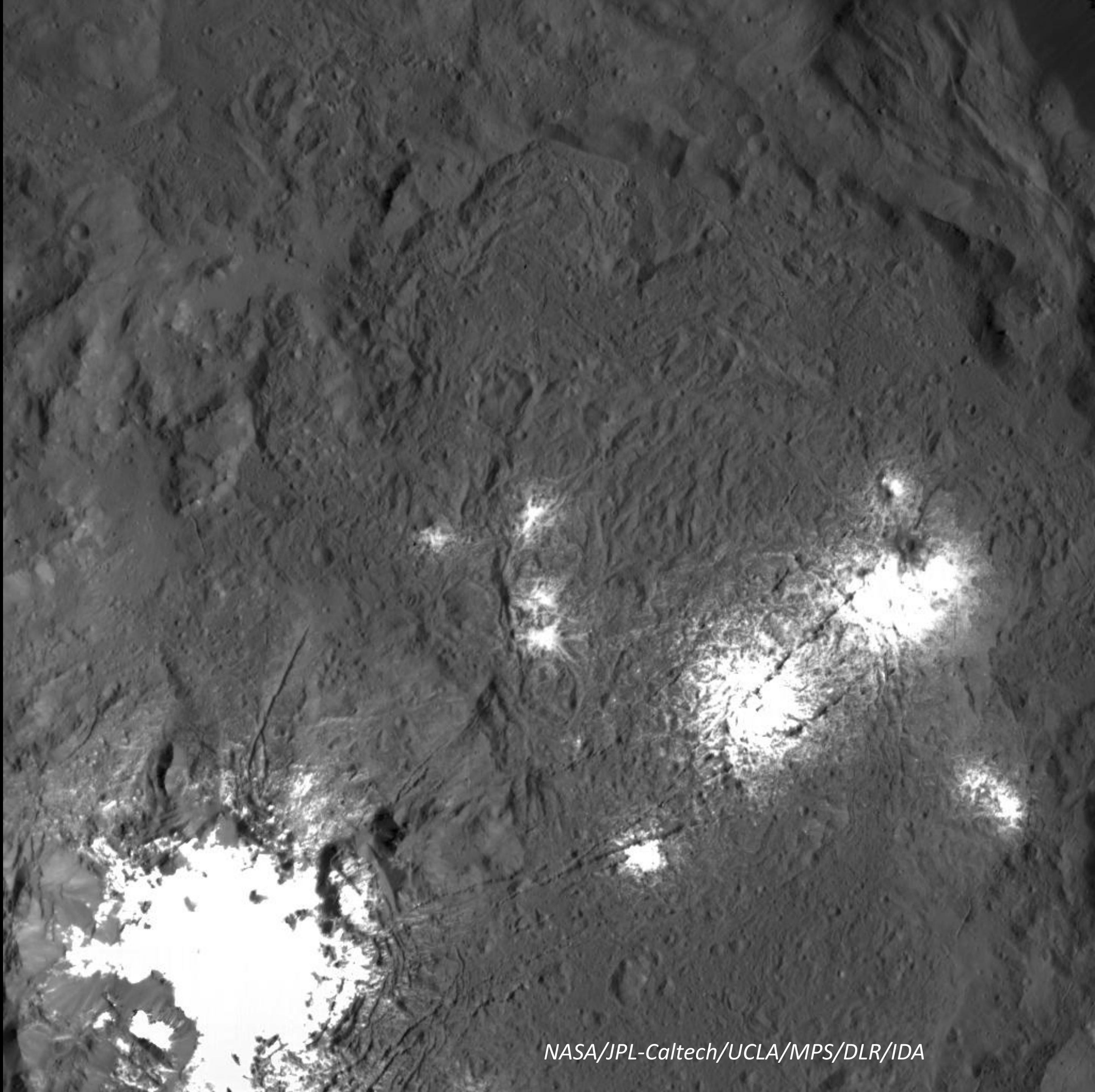

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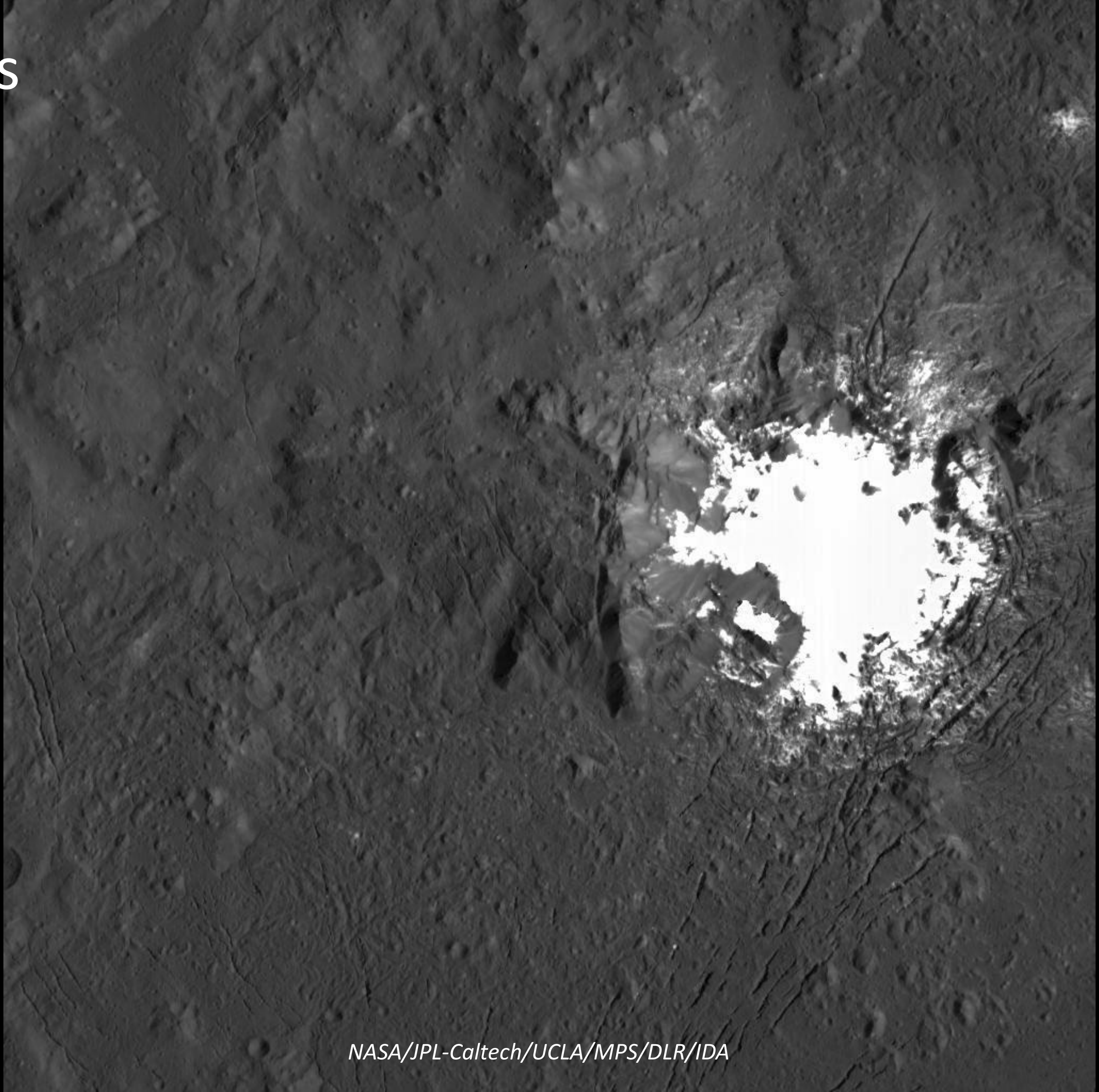

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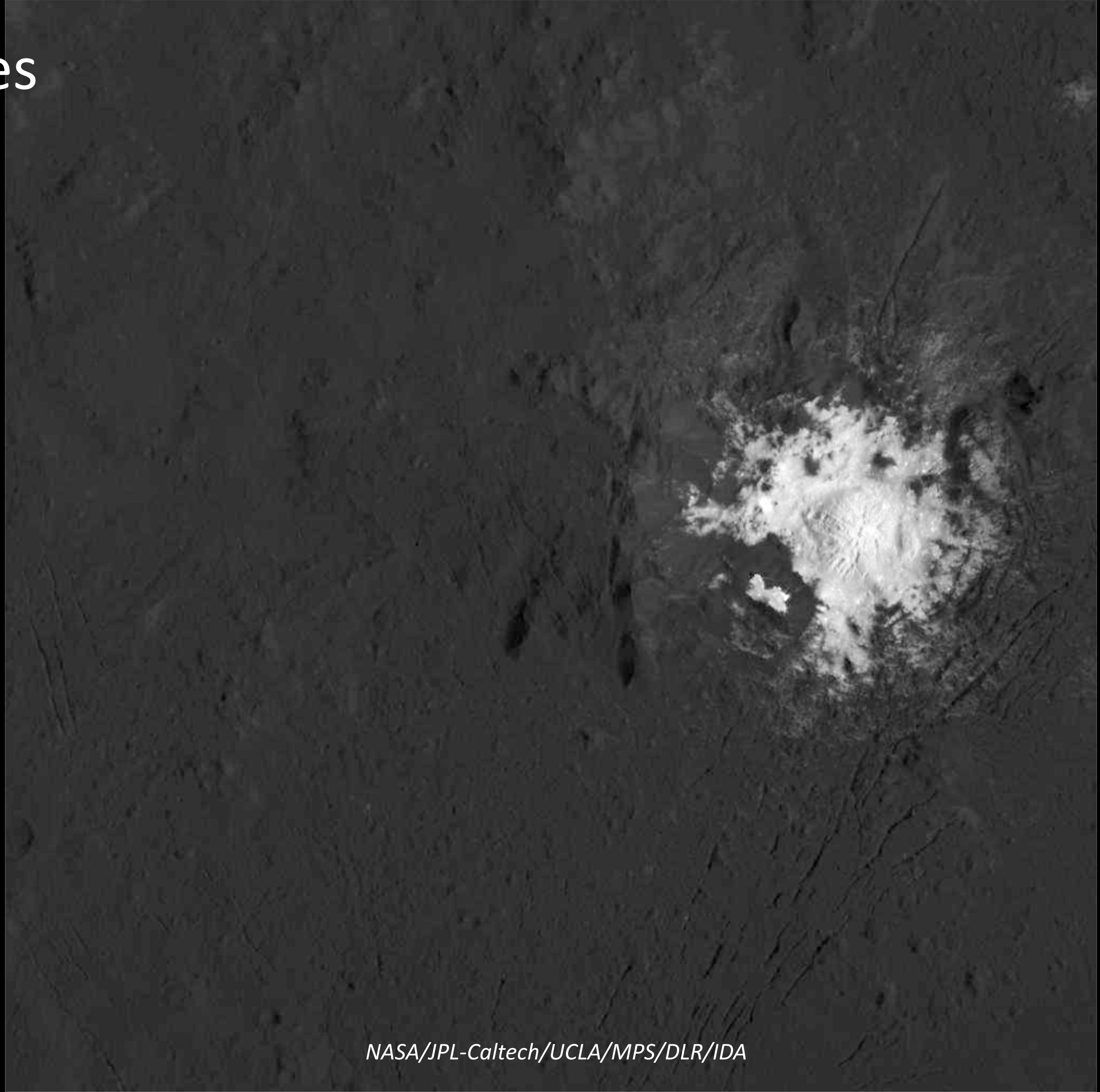
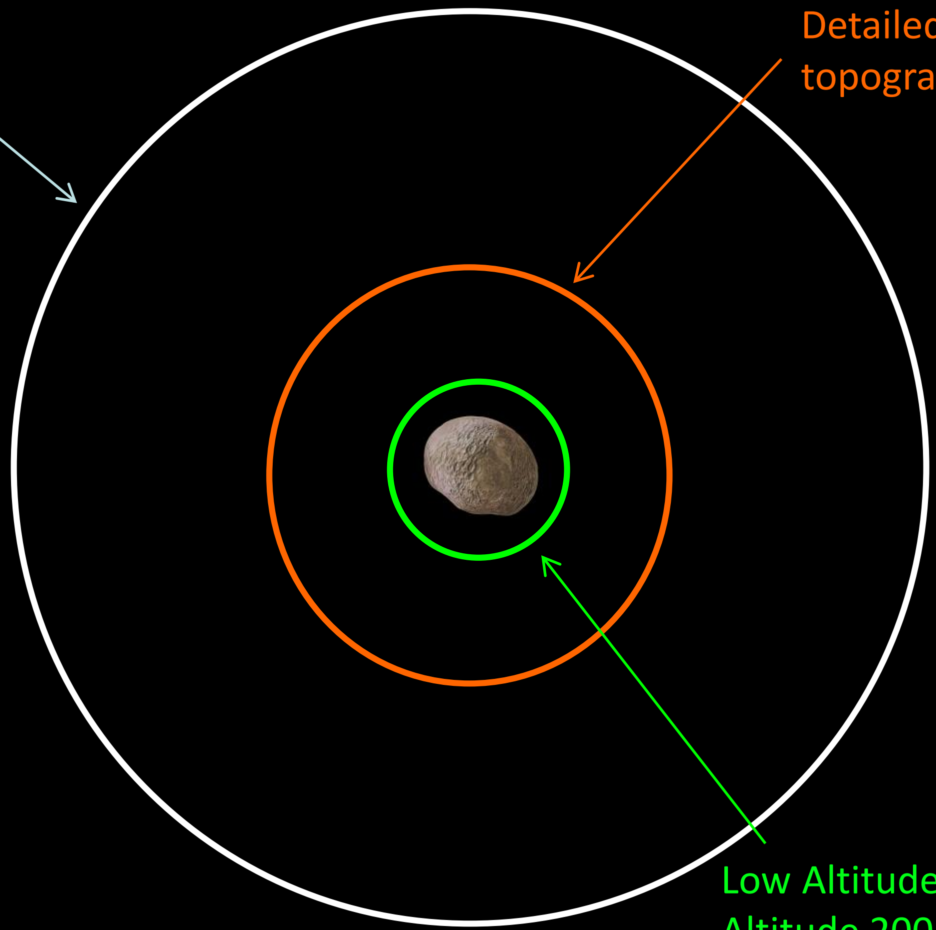


Science Orbits at Vesta:

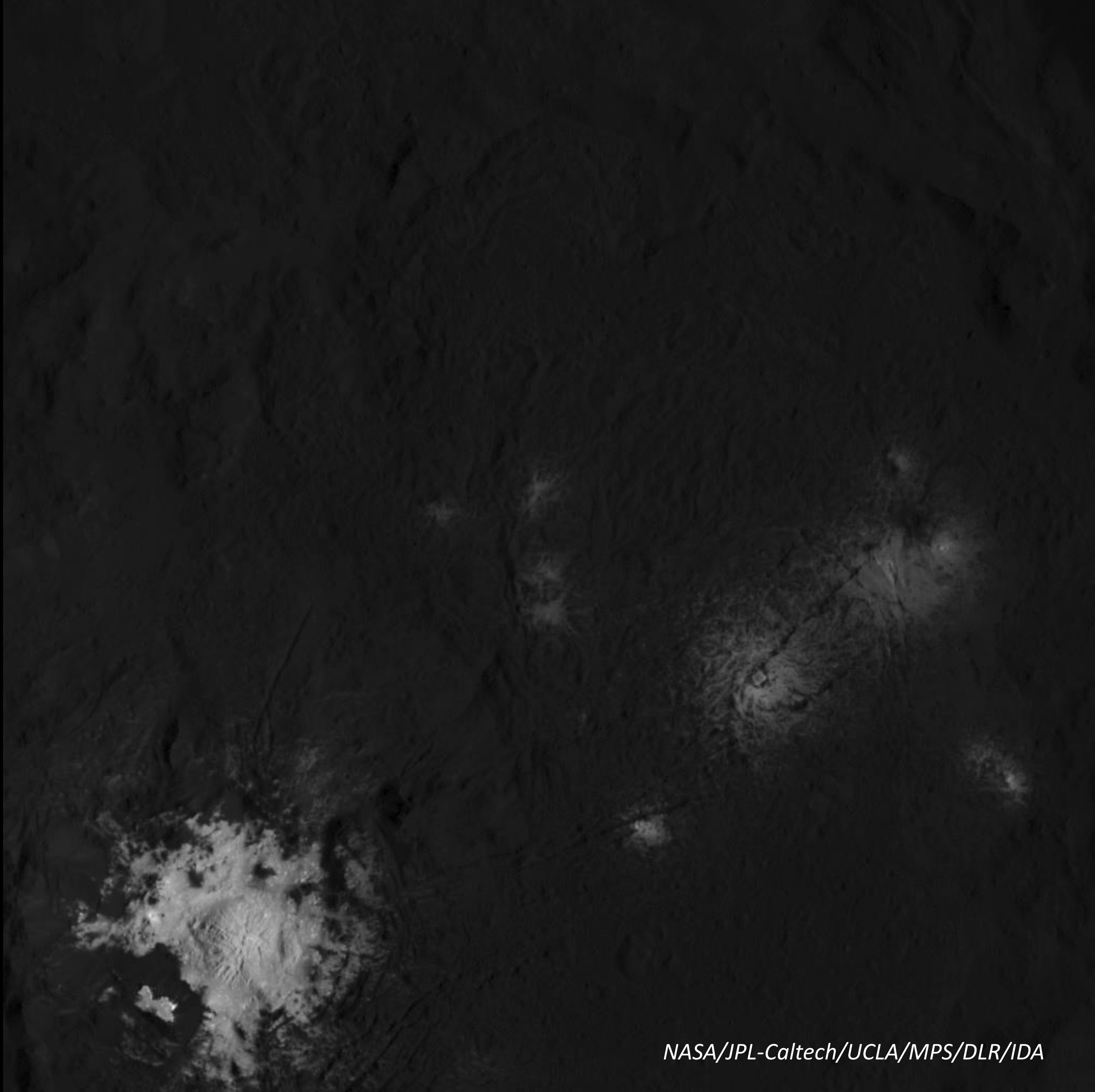
Survey Orbit:
Altitude 2730 [km]
August 2011
Look for Moons,
dust, First global
map

High Altitude Mapping Orbit:
Altitude 680 [km]
October 2011 and June 2012
Detailed spectral maps and
topography



Low Altitude Mapping Orbit:
Altitude 200 [km]
December 2011-August 2012
Gamma ray and Neutron
Spectroscopy and Gravity Science

Ceres



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Basic Structure of Optimal Control Problems

Start: state $X(t=0)$

Basic Structure of Optimal Control Problems

Start: state *X(t=0)*

You make a control decision *v(0)*

Basic Structure of Optimal Control Problems

Start: state $X(t=0)$

You make a control decision $v(0)$

This makes the state evolve to $X(t=\Delta t)$

Basic Structure of Optimal Control Problems

Start: state *X(t=0)*
You make a control decision *v(0)*
This makes the state evolve to *X(t=Δt)*
You make a control decision *v(Δt)*
This makes the state evolve to *X(t=2Δt)*

Basic Structure of Optimal Control Problems

Start: state $X(t=0)$

You make a control decision $v(0)$

This makes the state evolve to $X(t=\Delta t)$

You make a control decision $v(\Delta t)$

This makes the state evolve to $X(t=2\Delta t)$

•

•

•

You make a control decision $v((n-1)\Delta t)$

End: This makes the state evolve to $X(t_f)$

Basic Structure of Optimal Control Problems

Start: state $X(t=0)$

You make a control decision $v(0)$

This makes the state evolve to $X(t=\Delta t)$

You make a control decision $v(\Delta t)$

This makes the state evolve to $X(t=2\Delta t)$

•

•

•

You make a control decision $v((n-1)\Delta t)$

End: This makes the state evolve to $X(t_f)$

Goal: find controls v(t) that optimize some objective involving the states, controls, and time

Optimal Control Problems Versus Feedback

Local Feedback: you simply react to the current state to decide the control (example keeping your car in the center of the lane)

We have to do this in space flight also! Though when we steer back into the "space lane" we actually solve a smaller optimal control problem to do so.

Optimal control: has a time global view solving for all controls that together maximize/minimize an objective (example: choosing the best arrangement of roads to take to get from one place to another to minimize trip time)

We solve optimal control problems in spaceflight to define the roads we follow in space: ***Reference Trajectory***